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INTERNATIONAL COMPARISO
OF COST OF LIVING

A STUDY OF CERTAIN PROBLEMS
CONNECTED WITH THE MAKING OF INDEX NUMBE
OF FOOD COSTS AND OF RENTS

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PREFACE

The International Labour Office has always given considerable attention to the problem of international cost-of-living comparisons, both in its theoretical and practical aspects, and various articles were published in the *International Labour Review* from 1924 onwards, in which tentative comparisons based on the prices of foodstuffs were made, and the difficulties met with in dealing with the problem were discussed. In 1930, the Office was requested to make an enquiry into the relative costs of living in certain European countries compared with that in Detroit (U.S.A.) and the Statistical Section was entrusted with the task. The results were published in a special study in 1932[1]. Although this enquiry was limited to a particular case of this problem, viz. the amount which should be paid to a worker in certain European cities in order that he may have a standard of living approximately equivalent to that of a certain category of American worker of given income, yet the enquiry raised many problems of method which are discussed in the study[2]. In 1931, a conference of official labour statisticians was held at the International Labour Office, which discussed, among other things, the general question of international comparisons of cost of living. This conference made certain recommendations concerning the future work of the Office on these matters[3] and the present volume gives the first results of the studies undertaken in this field by the Statistical Section of the Office, under the direction of Mr. J. W. Nixon.

Part I of this volume relates solely to food costs and was prepared by Mr. H. Staehle, Member of the Statistical Section. The principal theoretical difficulty of comparing the relative costs of food in different countries (apart from practical difficulties such as the absence of adequate data for many countries) is that habits of consumption differ—in some cases widely—from country to country. The calculation of comparative food costs by means of a standard or fixed budget, as used by the Office in its first enquiries, was not satisfactory. This was fully realised by the Statistical Section in its study of 1932 referred to above, and account was taken in each comparison of a European city with the American city both of the consumption habits of the city (or country) in question, and of those of the United States. The Conference of Labour Statisticians of 1931 recommended " that comparisons cannot usefully be made between countries of widely differing habits and customs; and that the closer are the consumption habits in different countries, the more trustworthy are the comparisons likely to be. The calculation of index numbers by the use of information as to working-class consumption furnished by family budget enquiries should be made not only as hitherto on a fixed international budget but also on the basis of various national or regional (i.e. groups of countries) budgets ". In this Part, the author has analysed the available data on food consumption as given in family budget enquiries and in retail price statistics, with a view to arriving at " regions of good comparability ". The conditions for satisfying this criterion are mathematically analysed and examples are given of the grouping of countries (or cities) into regions in which habits of consumption are sufficiently " close " (to quote the term of the Conference of Statisticians) as to render comparisons reliable.

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[1] INTERNATIONAL LABOUR OFFICE: *A Contribution to the Study of International Comparisons of Costs of Living. An enquiry into the cost of living of certain groups of workers in Detroit (U.S.A.) and fourteen European towns.* Studies and Reports, Series N (Statistics), No. 17. Geneva, 1932.

[2] See in particular Chapter II, Parts I and II (pp. 7-27) and Appendix IV, " Further Observations on the Methods and Validity of the Results of the Present Enquiry ", pp. 216-246.

[3] The text of the recommendation and a discussion of the work of this Conference will be found in the *International Labour Review*, Vol. XXIV, No. 1, July 1931 (" The Fourth Conference of Labour Statisticians "), also in *The International Standardisation of Labour Statistics*, Studies and Reports, Series N, No. 19.

Part II relates to comparisons of rents and was prepared by Mr. R. Guye, Member of the Statistical Section. The problem of comparing working-class expenditure on rents in different countries is rendered especially difficult by variations in the standard of accommodation obtained in return for the rent payment, e.g. the number of rooms per dwelling, the provision or absence of a bathroom, etc., must be taken into account in international comparisons. The Conference of Labour Statisticians in 1931 recommended " in view of the desirability of including, in the data relating to cost of living, information as to the level of rents in each country ", that the Office should collect "information with regard to the average rents of the predominant types of working class dwellings; together with a description of the accommodation provided and of the general character of the dwellings covered. The International Labour Office should study the possibility of obtaining one or more standard units of housing for the various purposes of international comparison ". The Statistical Section, in accordance with this recommendation, prepared a form of enquiry on the subject of working-class rents; the material received has been published in the *International Labour Review.* In Part II of this volume, this material is analysed from the point of view of obtaining international indexes of the cost of rent. After a theoretical study of the problem, a discussion of the best " units of housing " for this purpose, and of the various methods of averaging and of weighting, practical examples are given of the results obtained by applying different criteria.

The problems discussed in this volume are only a part of the problems inherent in the comparison of the cost of living as a whole. Not only do other items remain to be dealt with—e.g. heating and lighting—but there remains the problem of combining the indexes of these various groups of expenditure. It is hoped that the contributions towards that problem contained in this volume will mark an important step towards the solution of international comparisons of cost of living.

CONTENTS

Page

PREFACE III

PART I

International Comparison of Food Costs

By H. STAEHLE

I. *Introduction* 1

Formula recommended by Professor Fisher 1
The Principal Difficulty of Application 3
Three Attempts to Overcome the Difficulty 4

Bowley's Attempt (1919) 4
The Chain System 6
The Circular Test 7

II. *Theory of the Comparability of Prices* 9

Necessity for a Compromise 9
Conditions of Good Comparability 10

Form and Method of Calculating Criterion of Comparability 10
Analysis and Interpretation 12
The Influence of the Standard of Living on Comparability 19
Summary and Conclusion of Theoretical Considerations. Solution of a Last Difficulty 29

III. *Formation of Regions of Good Comparability* 31

Preliminary Questions of a Practical Nature 31

Practical Solution 31
Choice of the Countries observed and the Date of Comparison 32
Data Used 32
Monetary Units 35

Structure of the Regions 35
Choice of the First Centre: Stockholm or Oslo 38
Composition of the First Region 43
Composition of the Other Regions 47
Various Special Cases 55

IV. *Establishment of Final Indexes for July 1930* 60

Choice of Formulae for Current Comparisons 60
Results obtained by Applying the Chosen Formulae 62

V. *Conclusions* 72

APPENDICES:

I. Various Theories on the Relation between Price Index Numbers and the Standard of Living 74
II. Description and Sources of the Data Used 93
III. Data Used in the Calculations 96
IV. Composition of Regional Budgets 104

PART II

International Comparison of Rents

By R. GUYE

Page

I. *Introduction* 107
II. *Theoretical Examination* 111
  Determination of Standard Units of Housing 114
  Basis of Weighting 119
  Methods of Calculation 124
    Variable Weights 125
    Standard Weights 126
    Double Weights 126
    Mixed System 127
III. *Practical Application* 129
  The Statistical Material Used 129
  Experimental Calculations 132
  Examination of Results 139
IV. *Conclusions* 144

PART I

INTERNATIONAL COMPARISON OF FOOD COSTS

By H. STAEHLE

I

INTRODUCTION

The present study deals solely with the international comparison of food costs, and its object is to develop a reliable and at the same time practicable method of comparing simultaneously a certain number of countries. It is to some extent inspired by the resolutions of the fourth International Conference of Labour Statisticians (1931), and is also based on the work done by the Office in connection with an international enquiry into comparisons of costs of living—and therefore, of food costs as well—in a certain number of European towns with the cost in Detroit (U.S.A.) [1].

FORMULA RECOMMENDED BY PROFESSOR FISHER

In that enquiry the comparisons of food costs were effected by means of a formula recommended by Professor Irving Fisher especially, as well as by other authors both before and after him. If the comparison is to be made between two countries (or dates) 0 and 1 (0 being taken as base), and if for a certain number of articles of food in the two countries (or at the two dates) the retail prices are p_0', p_0'', p_0''' ... and p_1', p_1'', p_1''' ... and the quantities consumed per family or other consumption unit in the unit of time are $q_0', q_0'', q_0''' \ldots$ and $q_1', q_1'', q_1''' \ldots$, then the formula may be written as follows:

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[1] Cf. INTERNATIONAL LABOUR OFFICE: " A Contribution to the Study of International Comparisons of Costs of Living ". Studies and Reports, Series N, No. 17, hereinafter referred to as " Contribution ".

$$\text{(i)} \qquad \text{Index} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \cdot \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}\,.$$

It is therefore the geometric mean of two ratios, namely

$$\text{(ii)} \qquad \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = P_0$$

$$\text{(iii)} \qquad \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = P_1$$

which are themselves index numbers—that is to say, weighted averages of the price ratios $\left(\frac{p_1}{p_0}\right)$[1].

The Office adopted this formula for the following reasons:

(1) The results can be presented in terms that exactly correspond to the problem of determining the relation between equivalent incomes—that is to say, incomes capable of procuring the same total economic satisfaction or the same standard of living.

(2) It takes into account existing differences in the customs of the different countries without, however, being unduly affected by the particular standard of living of these countries or by the date of the budget enquiries used to provide weights.

(3) Provided that the articles considered in the calculation are in fact all of them consumed—that is to say that data are available concerning the quantities consumed in the two countries—the values of P_0 and P_1 are in practice not affected by the number of these articles. This observation is clearly valid only for a relatively limited interval. It would not justify, for instance, a conclusion that it would be sufficient to make a comparison of one price only.

(4) The result is independent of the choice of base, the index established on base 0 being the reciprocal of the index established on base 1.

These considerations led the Office to conclude that it will be sufficient to have an adequately representative and not too out of date food budget for each country concerned in order to obtain a reliable result, by applying formula (i) to a number of foodstuffs that are known to be consumed in both cases, the quantities consumed being also known [1].

---

[1] The first average (P_0) is due to Laspeyres, and the second (P_1) to Paasche. The average of the two was proposed by Bowley and Walsh long before Fisher, though the merit of laying special stress on it belongs to the latter.

THE PRINCIPAL DIFFICULTY OF APPLICATION

This conclusion thus leads to a whole programme for the establishment of international cost-of-living index numbers. But it must not be overlooked that the aim of the enquiry which led to this conclusion was somewhat different from that of the proposed periodically compiled and widely ramified international cost-of-living (or food) index numbers. The computations in question were in fact undertaken to determine, for a certain number of towns, the incomes which, having regard to differences in prices, tastes and habits, would be equivalent to a given income in Detroit. In this case, the unknown quantities were thus the incomes themselves, and the cost-of-living indices only derived indirectly from these calculations in that these equivalent incomes once determined, their ratios one to another furnished cost-of-living indices. On the other hand, the indices which we propose to establish in this memorandum should serve mainly to determine the equivalence or non-equivalence of incomes of stated amounts. The unknown quantity in the present case is no longer the amount of these incomes but their comparative purchasing power. Leaving aside for the moment the logical complexities resulting from this difference between the present enquiry and the enquiry referred to, it may be noted that it was perfectly permissible for the purpose of the earlier enquiry to verify that the indices obtained from formula (i) were practically invariable in respect of different incomes irrespective of any simultaneous variation in the quality of the comparison. Thus, it was possible to leave out of account fluctuations in the difference between P_0 and P_1, according as index (i) related to different pairs of towns, or, for each pair, to different levels of income. But, as will be seen, that difference is of decisive importance in any price comparison, whenever the numerical value of the index is no more the only object, but attention is paid also to the degree of confidence that can be placed in it.

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[1] Cf. " Contribution ", p. 229.

As was shown in the report of the Office on the above-mentioned enquiry [1], the values of P_0 and P_1 may in certain circumstances be conceived as indicating the upper and lower limits between which the " true " cost-of-living ratio must lie—that is to say, the ratio between equivalent nominal incomes in two countries. This is in itself sufficient to suggest that if the divergence between the two extreme values is too great, the problem remains ill-defined.

The conclusion is in fact a well-known one. Wicksell, for instance, wrote: " If these two calculations (of the averages P_0 and P_1) result in values that are identical or very close to each other . . . there is reason to consider the result as finally correct. If, on the other hand, the values obtained are essentially different, the matter must be left at that. . . . In practice, it is true, an average can be taken of the two different values, but the significance of this average will be purely conventional. It is the very nature of the case that appears to prevent further progress" [2].

Several other authors could be cited. They all have this in common, that they tend to give up the attempt.

THREE ATTEMPTS TO OVERCOME THE DIFFICULTY

Only three attempts have in fact been made to overcome the difficulty: one due to Bowley, who dealt with the question in 1919; the second proposed by Lehr in 1885 and by Marshall in 1887 (the chain system); and, finally, the third put forward by Westergaard and Walsh (the circular test). They are worth considering briefly.

(a) *Bowley's Attempt (1919)*

After noting that the method consisting in calculating P_0 and P_1 appears to be valid when the habits of living and the commodities purchased are not dissimilar, Bowley [3] arrives at the conclusion that it breaks down as between, say, India and England, or as between 1815 and 1914, and possibly between 1914 and 1919. Nevertheless, he proposes for cases of an enforced change in actual consumption (he is thinking of the regulations resulting from war conditions) a method that could naturally be applied also where the change is due to other reasons. It may therefore be regarded as an attempt to solve the problem of too great a divergence between P_0 and P_1, since it is a matter of indifference whether the change is enforced or voluntary.

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[1] Contribution ", pp. 220-221.

[2] Knut WICKSELL: *Vorlesungen über Nationalökonomie*, Theoretischer Teil, Vol. 2, p. 155, Jena, 1928.

[3] BOWLEY, Arthur L.: " The Measurement of Changes in the Cost of Living ", in the *Journal of the Royal Statistical Society*, 1919 (New Series), Vol. LXXXII, pp. 343-361, and in particular pp. 348 and 349.

The new method is the following: if consumption has changed in character between the dates 0 and 1, the average variation in quantities may be expressed by $\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0}$ (or a weighted index of the ratios $\frac{q_1}{q_0}$). This average variation would at the same time indicate the proportion in which the consumer's " satisfaction " has varied as a function of the change in his consumption. If it has diminished, for instance, in the ratio of 100 to 80, it will be sufficient to increase the expenditure at date 1 in the ratio 100 to 125 in order to obtain a sum representing at the date concerned approximately the same satisfaction as that represented by the actual expenditure at date 0. The ratio between these two sums (or equivalent expenditures) will give the price index. We may therefore write:

$$\Sigma p_1 q_1 \cdot \frac{\Sigma p_0 q_0}{\Sigma p_0 q_1}$$

for the equivalent expenditure at date 1, and

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \cdot \frac{\Sigma p_0 q_0}{\Sigma p_0 q_1} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$$

for the index to be applied when there has been a considerable change in consumption. This is in fact formula P_1.

It may be objected to this method that there is clearly no reason —especially when the comparison is not between two dates, but between two countries 0 and 1—why the variation in " satisfaction " should be measured by the ratio $\frac{\Sigma p_0 q_1}{\Sigma p_0 q_0}$— that is to say, by weighting the ratios $\left(\frac{q_1}{q_0}\right)$ by the expenditure at date 0[1]. It would be just as possible to take $\frac{\Sigma p_1 q_1}{\Sigma p_1 q_0}$, an expression weighted by expenditure at date 1. This would give

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[1] Bowley writes himself, while speaking of the index numbers (ii) and (iii), in his *Elements of Statistics*, 4th edition, London, 1920, p 211: " There is nothing in general to choose between (ii) and (iii) but . . . one year has the same claim to be included as the other. . . ."

$$\Sigma p_1 q_1 \cdot \frac{\Sigma p_1 q_0}{\Sigma p_1 q_1}$$

for the equivalent expenditure at date 1, and

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \cdot \frac{\Sigma p_1 q_0}{\Sigma p_1 q_1} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = P_0$$

for an index which, according to Bowley's argument, would be as applicable as P_1 when there have been considerable changes in consumption. This brings us back to the starting point, and the method advocated by Bowley in 1919 therefore does not seem to have solved the problem [1]. Hence, we may refrain from discussing its utilitarian implications, which are in no way admissible, without additional hypotheses, at least in the form given by Bowley in his article.

(b) *The Chain System*

Concerning this second attempt, by which prices in two countries are compared indirectly when their direct comparison gives too wide a divergence between P_0 and P_1, it should be remembered that the method was originally proposed to allow for modifications in the list of articles to be included for a chronological index. The idea was to establish a comparison between dates 0 and 2 by means of introducing an intermediate date 1, at which consumption is assumed to diverge less from the consumptions at dates 0 and 2 than the divergence between these two. The idea of applying this principle to comparisons in space has been accepted by certain writers.

It may be objected, however, that the method introduces a very arbitrary element even in chronological comparisons, for while it may be possible to find an intermediate date 1 between dates 0 and 2, it is just as possible, at least theoretically, to find others situated between 0 and 1 and between 1 and 2. There is no objective criterion by which to determine how far this procedure of intercalation should be carried. Moreover, it would be theoretically possible to compare two dates in this way even though no article consumed at the first date enters into consumption at the last date—an obviously absurd result, which, though very rare in chronological comparisons (unless they cover several centuries), may quite well occur in a comparison between widely separated countries.

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[1] It should be observed that Bowley now prefers another method to the one here discussed. See his " Notes on Index Numbers ", *Economic Journal*, Vol. XXXVIII, 1928, pp. 216-237, and in particular, pp. 228-230. See also Appendix I, pp. 81-83 below. He himself now considers that the two expressions for the change in standard of consumption are of equal value, but no longer uses them to deduce price index numbers.

In the case of inter-local comparisons there is yet another serious difficulty, namely the choice of the intermediate countries or localities. Strictly speaking, the problem arises, although less obviously, also for chronological comparisons. It is even possible that consumption at a date 2 subsequent to date 1 or previous to date 0 may be intermediate between the consumptions at dates 0 and 1 which it is wished to compare. In this case the strict application of the principle of intercalation on the basis of similarity of consumption should involve a roundabout passage by date 2 in order to compare 1 and 0.

It may be stated, however, that this problem arises more particularly in inter-local comparisons, and the question is whether there is any criterion for an objective determination of the intermediate localities. This problem was discussed at the fourth Conference of Labour Statisticians, which replied in the negative [1].

It may be added that the above objections to the chain system in chronological comparisons, arising out of the necessarily arbitrary nature of the intercalation, apply also to inter-local comparisons, thus rendering the result all the more problematical. It seems, therefore, very doubtful whether the system can be used as a general method for the simultaneous comparison of food costs in a large number of countries.

(c) *The Circular Test*

Strictly speaking, this is not a special method of comparison, but simply one property of certain formulæ which has been postulated as indispensable, in particular by Westergaard and Walsh, but rejected by Fisher. This property may be described as follows:

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[1] Resolution adopted by the Conference, paragraph 15. Cf. *International Labour Review*, Vol. XXIV, No. 1, July 1931, p. 19.

The circular test is satisfied by any formula which yields the *same result for the comparison of two countries, whether direct or indirect, provided that the list of articles considered is the same in all countries entering into the calculation.* This condition is satisfied only by a very small number of formulae. Thus, formula (i) above does not satisfy it. On the other hand, it is satisfied by the formula adopted by the Office for its current comparisons, namely:

$$\text{(iv)} \qquad \text{Index} = \frac{\Sigma p_1 q}{\Sigma p_0 q}$$

where q stands for fixed quantities consumed, for

$$\frac{\Sigma p_1 q}{\Sigma p_0 q} \cdot \frac{\Sigma p_2 q}{\Sigma p_1 q} = \frac{\Sigma p_2 q}{\Sigma p_0 q} \, .$$

It will at once be seen that this last èquation is valid only when the figures relate to the same articles.

It is obvious that any formula satisfying the circular test—formula (iv), for instance—will thus enable apparently quite satisfactory comparisons to be obtained even when the value of P_0 and P_1 are very different. It will even be impossible to decide whether a comparison is good or bad, for, from the point of view of form, it could never be better. Only in cases where the q's in formula (iv) are calculated as $\left(\frac{q_0 + q_1}{2}\right)$ and where the resulting basis of comparison is too limited (for lack of a sufficiently wide common consumption), would it be possible to doubt the excellence of the results. Far from solving the problem that arises when P_0 and P_1 are too different, the application of formulae satisfying the circular test is merely a way of evading the difficulty.

II

THEORY OF THE COMPARABILITY OF PRICES

Necessity for a Compromise

The above brief discussion has thus led to the following result. Neither the chain system nor a formula satisfying the circular test can be adopted purely and simply as a general method for comparisons of food costs. Furthermore, the difficulty due to too wide a divergence between P_0 and P_1 remains untouched. It must therefore be recognised that the problem can be solved only by way of a compromise, that is to say, by giving up direct comparisons when the divergence is too great.

This conclusion naturally leads to an idea already contained in the first international comparisons of food costs undertaken by the Office and further developed in the resolution of the fourth International Conference of Labour Statisticians[1]. This is the idea of limiting (at least as a first approximation) the *direct* comparisons to certain " regions ", or groups of countries or towns, between which these comparisons are reliable. It is immediately clear that precisely the divergence between the values of P_0 and P_1 will serve as a means for trying to group the countries or towns for which sufficient data are available into " regions " in the above sense. It is self-evident that the use thus made of formulae (ii) and (iii) in no way implies as yet that formula (i) should finally be adopted for current comparisons. It should be noted, however, that it is only by means of the divergence between P_0 and P_1 that it will be possible to judge of the quality of a particular comparison, and this possibility does not exist when formulae such as (iv) are applied.

As to the comparison of localities belonging to different regions, between which comparisons are therefore less good, this second stage presents special problems.

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[1] Cf. *loc. cit.:* Resolution adopted by the Conference, paragraph 13.

The points for further consideration may thus be summed up as follows:

(1) Analysis of the criterion to be applied for forming " regions of direct comparison ";
(2) Principles for its application;
(3) Choice of the method for direct comparisons between localities belonging to the same region;
(4) Choice of the method for comparisons between localities belonging to different regions.

CONDITIONS OF GOOD COMPARABILITY

As was stated above, the " regions " should comprise countries and towns between which direct and reliable comparisons are possible. These will be " regions of good comparability ". For the criterion of comparability, it also follows from the above that the divergence between P_0 and P_1 will serve as a starting point. Before proceeding, however, to practical applications, it will be necessary to consider exactly whether the difference between these two averages may in fact be taken as an indication of the comparability of prices in the two countries concerned.

A. — *Form and Method of Calculating Criterion of Comparability*

To this end the divergence between P_0 and P_1 will be considered in the following form:

$$D = P_1 - P_0 \tag{v}$$

By expressing the difference between P_1 and P_0 in terms of P_0 the values of D become directly comparable from one case to another. Subject to the provisos contained in the following analysis, the smaller the value of D, the more comparable will two countries be. It may be noted here that this criterion does not show whether a particular comparison is bad or good; but it makes it possible to choose the best among a number of comparisons of a particular country with any number of other countries. It therefore enables us to decide with what other country a particular country compares best. It will be found in fact that in certain cases values of D that are relatively high will have to be considered sufficient simply because D would be even greater if the country in question were compared with any other country.

In addition, the only case considered will be that in which D is calculated in such a way that the list of articles used for establishing P_0 on the one hand and P_1 on the other is the same. This condition follows from argument (3) given on page 2. It has been shown [1], although only empirically, that values are obtained for P_0 that may vary very considerably when the list of articles is extended beyond the consumption common to countries 1 and 0, while it is remarkably stable if only the articles common to the two are considered. It is clear that the criterion of comparability D should therefore be prevented from being too much affected by the number of articles serving for its calculation, by limiting its application to articles of common consumption.

In view of the importance of this argument, it needs to be justified by a rather more general development than that with which it was presented. It should be noted that for establishing the ratio P_0 it is sufficient to know for each article the prices in the two countries and the quantity in the base country. Further, for calculating the ratio P_1 it is sufficient to know for each article the two prices and the quantity consumed in the second country. Now, the price data are generally more accessible than the quantity data, so that one might be tempted to calculate P_0 on the basis of a list of articles differing from that contemplated for the calculation of P_1. But this procedure would increase the precision of the two averages P_0 and P_1 only on condition that the distribution of the weighted variables conforms to the law of error. It therefore becomes necessary to consider what the chances are that this is so.

It will be remembered that P_0 and P_1 are weighted averages of the price ratios $\frac{p_1}{p_0}$. Now, the variations of p_1 may be considered as independent of the variations of p_0. There can in fact be no close relations between the prices of the same article in two different countries. Each $\frac{p_1}{p_0}$ ratio may therefore be taken as an observation relating to the same entity, namely, the ratio of prices between the two countries. But it cannot be assumed that these various " observations " have a distribution conforming to the law of error except for the $\frac{p_1}{p_0}$'s which relate to articles of common consumption. Only in this case can we expect there to be no biased error, and therefore a fundamentally symmetrical distribution. On the other hand, when the calculation of P_0 is extended to articles for which only the prices and not the quantities consumed in country 1 are known, the chances are high that the errors will be biased. For the absence of consumption data for country 1 is probably due to the fact that the articles are not much consumed. It follows, therefore, that there is every chance that the prices of these articles in country 1 will be relatively high, that is to say, that $\frac{p_1}{p_0}$ will have values exceeding the average calculated on the basis of articles common to both countries. And the effect of including in P_0 articles whose price relatives suffer from biased error will be accentuated by the weighting. A similar line of reasoning applies also to the calculation of P_1. Consequently, the extension of the list of articles for the establishment of either P_1 or P_0 beyond the field of common consumption of the two countries will increase the difference between P_0 and P_1. But corresponding to this increase in D, there will be no increase in the precision of P_0 or P_1, nor will the comparability of prices in the two countries be less. It may be added that the following analysis will not apply in cases where P_0 and P_1 are obtained on the basis of a list of different articles.

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[1] " Contribution ", pp. 224-229.

B. — *Analysis and Interpretation*

It has been shown by many writers [1] that the values of P_0 and P_1 may be considered as indicating the upper and lower limits respectively between which the "true" price ratio should lie, if it can be assumed that the tastes and habits of the individuals concerned are the same in both situations 0 and 1. By the "true" price ratio is understood the ratio between equivalent nominal incomes. Concerning the identity of individual tastes, this can obviously be assumed only for comparisons between two fairly close dates in the same country and for individuals *belonging to the same social class*. It is therefore impossible to apply this reasoning as it stands to inter-local comparisons. On the other hand, it can be shown that in this latter case:

(1) P_0 can always be taken as representing the upper limit of the "true" price ratio from the point of view of individuals in country 0 and similarly P_1 for individuals in country 1 [2];

(2) the greater the similarity between the tastes of consumers in countries 0 and 1, the closer will P_0 and P_1 approach the lower and upper limits of the price ratio: P_1 will become the lower limit from the point of view of individuals in country 0,

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[1] See Appendix I, pp. 74-90 below.

[2] Cf. Haberler, *Der Sinn der Indexzahlen*. Tübingen, 1927, p. 95.

and P_0 the lower limit from the point of view of individuals in country 1.

If, therefore, to make use of the example given on another occasion[1], we have in a comparison between Detroit (0) and Berlin (1):

$$P_0 = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = 0.9 \text{ or } 90 \text{ per cent.}$$

$$P_1 = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = 0.8 \text{ or } 80 \text{ per cent.}$$

This merely means: (*a*) that for the American worker, prices in Berlin are lower by *at least* 10 per cent. as compared with prices in Detroit; and (*b*) that for the German worker, prices in Detroit are higher by *at most* 25 per cent. as compared with prices in Berlin. It is highly probable, however, that the American worker, by making slight adjustments to bring his manner of life nearer to the German, could reduce his expenditure in Berlin by an amount well above 10 per cent. of his expenditure in America, without in any way lowering his standard of living; and that the German worker, by making slight adjustments to bring his manner of life nearer to the American, could arrive at an expenditure in Detroit exceeding his expenditure in Germany by much less than 25 per cent. (or even falling below it) without in any way affecting his standard of living[2]. In other words, there is nothing to prevent such concessions to foreign customs from bringing down the price ratio from the American workers' standpoint (which at its maximum is $P_0 = 90$ per cent.), for instance, to 75 per cent., that is to say, a figure below P_1; while from the German workers' standpoint this ratio (which at its maximum is $P_1 = 80$ per cent.) might be, for instance, 95 per cent. in the most favourable case, that is to say, higher than P_0.

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[1] " Contribution ", p. 221.

[2] It may be of interest to quote the observation made by the British Board of Trade in 1908, in regard to its attempt to compare the cost of food in Germany and in England: " If then our English emigrant took to grey bread and coffee in place of white bread and tea, his total weekly expenditure would be only about 12 per cent. more than in England. But that is merely to say that if he lived more like a German his expenditure would be less ". It would clearly be inadmissible to try to use substitutions of this kind in international comparisons of costs of living, for there is no guarantee that the general well-being would not be affected. Cf. in this connection " Contribution ", pages 231-232.

It will thus be seen that the problem is not entirely solved, for the simple reason that it cannot be assumed, as would be possible for a comparison between two closely adjacent dates in the same country[1], that the tastes of the individuals considered are the same in the two countries. But the closer the similarity of tastes, the nearer does it approach solution.

Given this interpretation of P_0 and P_1, it is clear that in an inter-local comparison it is not sufficient for the values of P_0 and P_1 to be very close for obtaining a reliable comparison. On the contrary, supposing that the value of D does not vary directly with the dissimilarity in tastes, it must be concluded that the closeness between P_0 and P_1 makes it very probable that the "true" price ratio lies outside the interval of which P_0 and P_1 are the limits, the standpoint adopted being that of the consumer in either country. It must therefore be concluded that a small value of D is not in itself sufficient to guarantee good comparability of prices. For this it is also necessary that the tastes of the individuals considered in the two countries should not be too dissimilar. But is there a means of distinguishing in each particular case whether tastes are similar or not ?

To discover whether there is a reply to this question, it is necessary to find out what factors determine the value of D. For this purpose an analysis due to Bortkiewicz is given[2].

Assuming that

$$\frac{\Sigma wX}{\Sigma w} = M_1 \; ; \quad \frac{\Sigma wY}{\Sigma w} = M_2 \; ; \quad \frac{\Sigma wXY}{\Sigma w} = M_3$$

$$X - M_1 = x \; ; \quad Y - M_2 = y$$

$$\frac{\Sigma wx^2}{\Sigma w} = \sigma_x^2 \; ; \quad \frac{\Sigma wy^2}{\Sigma w} = \sigma_y^2 \; ; \quad \frac{\Sigma wxy}{\sigma_x \sigma_y \Sigma w} = r$$

the following relation may be established:

$$\text{(vi)} \qquad \frac{M_3 - M_1}{M_1} = r \cdot \frac{\sigma_x}{M_1} \cdot \frac{\sigma_y}{M_2} .$$

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[1] See on this subject Appendix I, pp. 87-88 below.

[2] L. v. BORTKIEWICZ, " Zweck und Struktur einer Preisindexzahl ", *Nordisk Statistisk Tidskrift*, Vol. 2, 1923, pp. 369-408; 1924, pp. 208-251. Bortkiewicz has given a more general and concise form to his analysis in his article: " Die Kaufkraft des Geldes und ihre Messung ", *Nordic Statistical Journal*, Vol. 4, 1932, pp. 13-14, from which formulae (vi) and (vii) are drawn.

Here r is the weighted coefficient of correlation between X and Y; σ_x and σ_y are the standard deviations of these variables, also weighted.

Now it is sufficient to make the following substitutions in these expressions:

$$X = \frac{p_1}{p_0}; \qquad Y = \frac{q_1}{q_0}; \qquad w = p_0 q_0$$

to obtain

$$M_1 = \frac{\Sigma p_0 q_0 \frac{p_1}{p_0}}{\Sigma p_0 q_0} = P_0; \qquad M_2 = \frac{\Sigma p_0 q_0 \frac{q_1}{q_0}}{\Sigma p_0 q_0} = Q_0$$

$$M_3 = \frac{\Sigma p_0 q_0 \frac{p_1}{p_0} \frac{q_1}{q_0}}{\Sigma p_0 q_0 \frac{q_1}{q_0}} = P_1$$

$$\frac{\Sigma w x^2}{\Sigma w} = \frac{\Sigma p_0 q_0 \left(\frac{p_1}{p_0} - P_0\right.}{\Sigma p_0 q_0} = \sigma_p^2,$$

$$\frac{\Sigma w y^2}{\Sigma w} = \frac{\Sigma p_0 q_0 \left(\frac{q_1}{q_0} - Q_0\right.^{2}}{\Sigma p_0 q_0}$$

$$\frac{\Sigma w x y}{\sigma_x \sigma_y \Sigma w} = \frac{\Sigma p_0 q_0 \left(\frac{p_1}{p_0} - P_0\right)\left(\frac{q_1}{q_0} - Q_0\right)}{\sigma_p \cdot \sigma_q \cdot \Sigma p_0 q_0} = r_{pq}$$

and therefore

$$\text{(vii)} \qquad \frac{M_3 - M_1}{M_1} = \frac{P_1 - P_0}{P_0} = r_{pq} \cdot \frac{\sigma_p}{P_0} \cdot \frac{\sigma_q}{Q_0} = D \cdot$$

Here D is our criterion of comparability[1].

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[1] D may also be analysed as shown below. If the following substitutions are effected in the above equations:

$$X = \frac{p_0}{p_1}; \qquad Y = \frac{q_0}{q_1}; \qquad w = p_1 q_1$$

D is therefore a function of three different elements[1]:

(1) the ratio of the weighted standard deviation of the price ratios $\left(\frac{p_1}{p_0}\right)$ and P_0;

(2) the ratio of the weighted standard deviation of the quantity ratios $\left(\frac{q_1}{q_0}\right)$ and Q_0;

(3) the weighted coefficient of correlation between the two variables $\left(\frac{p_1}{p_0}\right)$ and $\left(\frac{q_1}{q_0}\right)$.

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the result will be

$$M_1 = \frac{\Sigma p_1 q_1 \frac{p_0}{p_1}}{\Sigma p_1 q_1} = \frac{1}{P_1}\,; \qquad M_2 = \frac{\Sigma p_1 q_1 \frac{q_0}{q_1}}{\Sigma p_1 q_1} = \frac{1}{Q_1}$$

$$M_3 = \frac{\Sigma p_1 q_1 \cdot \frac{p_0}{p_1} \cdot \frac{q_0}{q_1}}{\Sigma p_1 q_1 \frac{q_0}{q_1}} = \frac{1}{P_0}$$

$$\frac{\Sigma w x^2}{\Sigma w} = \frac{\Sigma p_1 q_1 \left(\frac{p_0}{p_1} - \frac{1}{P_1}\right)^2}{\Sigma p_1 q_1} = \sigma p'^2\,; \qquad \frac{\Sigma w y^2}{\Sigma w} = \frac{\Sigma p_1 q_1 \left(\frac{q_0}{q_1} - \frac{1}{Q_1}\right)}{\Sigma p_1 q_1} = \sigma q'^2$$

$$\frac{\Sigma w x y}{\sigma x\, \sigma y\, \Sigma w} = \frac{\Sigma p_1 q_1 \left(\frac{p_0}{p_1} - \frac{1}{P_0}\right)\left(\frac{q_0}{q_1} - \frac{1}{Q_1}\right)}{\sigma p' \cdot \sigma q' \cdot \Sigma p_1 q_1} = r_{p'q'}$$

and hence:

$$\text{(vii}\,a\text{)} \quad \frac{M_3 - M_1}{M_1} = \frac{\frac{1}{P_0} - \frac{1}{P_1}}{\frac{1}{P_1}} = \frac{P_1 - P_0}{P_0} = r_{p'q'} \cdot \sigma p' \cdot \sigma q' \cdot P_1 \cdot Q_1 = D\,.$$

In this last equation, each factor has values different from those obtained from equation (vii), but we will not here enter into a detailed consideration of these differences. Our analysis will entirely be based on equation (vii), as developed by Bortkiewicz and presented in the text.

[1] Professor Bowley has pointed out that the use of the correlation coefficient in formula (vii) is arbitrary. For it can be shown that σ_p and σ_q are not involved, since:

$$D = r_{pq} \cdot \frac{\sigma p}{P_0} \cdot \frac{\sigma q}{Q_0} = \frac{\Sigma p_0 q_0 \left(\frac{p_1}{p_0} - P_0\right)\left(\frac{q_1}{q_0} - Q_0\right)}{P_0 \cdot Q_0 \cdot \Sigma p_0 q_0}\,.$$

It has not been possible, however, to give an economic interpretation of the elements of this simplified analysis. This is why it would not have lent itself to the development of the theory below, and why preference has been given to an analysis which, though artificial from the mathematical standpoint, can serve as a basis for economic understanding.

These various elements can be given an economic interpretation. As regards $\frac{\sigma_p}{P_0}$, this may be taken as a measure of the dissimilarity between the price systems in the two countries 0 and 1. For if the systems are identical $\left(\frac{p_1}{p_0} = \text{constant}\right)$, σ_p becomes zero. If, on the contrary, they are very dissimilar and the distribution of the ratios $\left(\frac{p_1}{p_0}\right)$ is very asymmetrical, σ_p may even reach values exceeding P_0. Similarly, $\frac{\sigma_q}{Q_0}$ may be taken as a measure of the dissimilarity between the consumptions of the two countries. The value can be zero where there is perfect proportionality between the quantities consumed $\left(\frac{q_1}{q_0} = \text{constant}\right)$ and σ_q can reach values exceeding Q_0 in the case of very asymmetrical distribution.

All that these two expressions indicate is whether there is more or less similarity between countries 0 and 1, and this separately for the price systems and the quantity systems. It is thus evident that by themselves they offer no means of judging of the similarity between the tastes of the individuals or families considered. For this, it would be necessary to be able to establish a causal connection (in one direction or other) between these two measures of dissimilarity, as being the only justification for thinking that there is a similarity in the economic reactions of individuals. But even were there a satisfactory correspondence between the deviations of the price ratios $\left(\frac{p_1}{p_0} - P_0\right)$ and the deviations of the quantity ratios $\left(\frac{q_1}{q_1} - Q_0\right)$ — which correspondence is measured precisely by r_{pq}, the third element of D — it would not be possible to distinguish whether it was due rather to the influence of prices on quantities (or of quantities on prices), or to the influence of differences between the real incomes of the families considered, or, finally, to other unknown and indeterminable influences. It should further be noted that even assuming that the tastes, real incomes and all the other circumstances of the individuals or families in the two countries are identical, the correlation will not be perfect unless the articles are completely independent (the consumption of each article thus depending only on its own price) and the elasticities of demand for the various articles are identical (although not necessarily equal to unity, to use Marshall's terminology).

It therefore seems useless to try to draw definite conclusions from the preceding analysis concerning the similarity of tastes of the individuals considered. This analysis makes it possible, however, to decide in cases where the value of D is relatively small whether this criterion can be accepted or not as indicating good comparability. It seems, for instance, that an almost complete absence of correlation, combined with fairly high values for the two dissimilarities, indicates bad comparability, even though the resulting D may be very small. For in this case the small value of D will be due solely to the fact that the two dissimilarities relate to different articles. The economic circumstances (prices and quantities) will none the less be different in the two countries, and it seems obvious that the absence of correlation cannot in this case be regarded as a guarantee of similarity in tastes. Further, positive correlation would indicate either marked dissimilarity in tastes or—and this is more likely—too wide a difference between real incomes[1]. It will therefore be necessary to lay down as a condition of good comparability that there must always be a certain negative correlation. It immediately follows that cases where this condition is fulfilled, but where the two dissimilarities are relatively large, must be considered as non-comparable, for the resulting D must necessarily be substantial[2]. Further, neither of the two measures of dissimilarity should approach zero, for in this case the correlation between the price ratios and quantity ratios would not indicate similarity in the economic reactions of the individuals considered. For this would in fact mean a wide dissimilarity in prices and a small one in quantities consumed, or else a small dissimilarity in prices and a wide one in consumption, and neither case appears to indicate a probability that the tastes and economic reactions of the individuals considered are particularly alike.

To sum up, the conditions of good comparability are as follows:

(1) D should have a relatively low value.

(2) There should be a certain negative correlation between $\left(\frac{p_1}{p_0}\right)$ and $\left(\frac{q_1}{q_0}\right)$.

(3) One of the two dissimilarities should not be very small when the other is relatively large.

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[1] See however the cases discussed later on, pp. 55-56.

[2] This case would indicate that individuals with generally similar reactions are placed in very different circumstances, a plausible reason for non-comparability.

(4) The two dissimilarities should not have too high values.

It is obvious that when making a practical application it will not always be possible to verify that all these conditions are fulfilled. A check may be made, however, whenever the choice in a particular case seems doubtful.

C. — *The Influence of the Standard of Living on Comparability*

Reference was made above to the possible influence of differences in real income or standard of living on the quality and result of price comparisons. There are three stages at which this influence may appear:

1. In the results of the price comparison, and in particular of applying formula (i).
2. In the variations that may take place in the value of D when the quantities q_0 and q_1 are replaced by quantities q'_0 and q'_1 representing the consumption of families with incomes differing from those consuming q_0 and q_1 in countries 0 and 1 respectively.
3. In the variations that may take place in the elements forming D, that is to say the weighted correlation between $\frac{p_1}{p_0}$ and $\frac{q_1}{q_0}$ and the two measures of dissimilarity $\frac{\sigma_p}{P_0}$ and $\frac{\sigma_q}{Q_0}$, as a function of variations in the incomes of the families in countries 0 and 1 to which the quantities q_0 and q_1 respectively relate.

The first form of influence—which shows itself in the results of applying formula (i)—has already been studied on the occasion of the Office enquiry cited above [1]. The object was then to ensure that the price comparison would not be affected by the particular standard of living of the families whose food consumption entered into the system of weights used for calculating formula (i). By means of a large number of checks the Office came to the conclusion that the result of applying the formula was practically independent of the incomes considered, and therefore of the standard of living. This was consequently an essential condition for that enquiry. It was at the same time sufficient, for it was enough to know that even if consumption data for families with a precisely equivalent income

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[1] " Contribution ", pp. 8-9, 222-224.

had been available (assuming that this equivalent income, the subject of the enquiry, was already known), the use of these data would have made practically no change in the result of the comparison (good or bad according as ($P_1 - P_0$) was large or small, but certainly not to be improved upon in the enquiry in question). This is not the place to reopen the question. It is mentioned solely because it formed the subject of certain criticisms in a memorandum by the Swedish Social Board.

The Swedish Social Board expressed its surprise that the Office had been satisfied in its enquiry with ascertaining that the result of applying formula (i) was independent of the standard of living of the families considered, leaving out of account the size of the difference between P_0 and P_1 [1] and the variations that changes in the standard of living might produce in this difference.

This latter point is precisely the second type of influence mentioned above, which the difference between the real incomes considered in the two countries may exert, namely the influence which appears in the variations of D. This is found, in fact, to be fairly large in certain cases, while it may be almost nil in others. If it had been found to be generally weak, this might have been considered sufficient, though an attempt should be made to generalise the reasons for its being so. But this not being the case, as will appear from tables I and II, the question arises whether it is indispensable for forming the " regions " that D should be constant, irrespective of the income class taken into consideration. By means of the analysis to which D has been subjected in the preceding pages it has been shown that its value is not a sufficient criterion of comparability, but that it is necessary to take into consideration the value of the elements composing it. It is precisely this fact which makes it useless to lay down as a condition of good comparability that D should be independent of the incomes considered and constant.

It should be observed that had D been found to be the only criterion of comparability, it would in fact have been indispensable for it to be constant. For otherwise this criterion would lead to the

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[1] It must be noted that the Office did not leave out of consideration the case where the difference between P_0 and P_1 is large, nor the possibility of arriving at better results through application of the chain-system. However, the Office was led to the conclusion that this method would have introduced a new element even more arbitrary than those already inherent to the method actually applied, since the number of countries included in the enquiry was small; in addition, it was impossible to include other countries only to be used as " bridges ".

> formation of "regions", no doubt comprising countries that compare well with each other, but this good comparability might be due simply to the particular systems of weights used for the calculation of D. By choosing for the same country a system of weights based on the quantities consumed by families with a higher or lower income (and therefore standard of living), other values of D might perhaps be obtained, leading possibly to the placing of a particular country in another region. In other words, it would be necessary for the comparability between the localities composing a region to be independent of the standard of living of the families considered [1].

This question may therefore be left out of account, and the third type of influence of the difference between the real incomes taken into consideration in countries 0 and 1 may be discussed, namely the influence appearing in r, $\frac{\sigma_p}{P_0}$, and $\frac{\sigma_q}{Q_0}$.

This question is attacked here in two ways. On the one hand, use is made of certain direct checks, by the calculation in certain cases of the values of the three constituent elements. As, however, this check necessitates much laborious calculation, we have also argued by analogy, using the results obtained for another measure of the dissimilarity of consumption as a function of the incomes considered, since its variations may be considered as similar to those of $\frac{\sigma_q}{Q_0}$.

For the direct checks two cases are given. The first makes use of the data for Sweden and Finland, where P_0 and P_1, and therefore D, are practically constant, irrespective of the income of the families whose consumption constitutes q_0 and q_1 in the calculations. The second relates to the comparison between Germany and Poland, where P_0 and P_1, as also D, vary fairly markedly with the income of the families whose consumption is used in the calculations. The results of these two tests are contained in tables I and II.

Three incomes have been chosen for each country considered. They represent the lowest, an intermediate (sometimes the general average), and the highest income group among all those for which the average quantities consumed are given in the budget enquiries. These quantities, taken as q_0 and q_1, were used to form the nine

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[1] As will be seen later on, this solution has been adopted in practice. Nevertheless, no attempt has been made to discover why D remains sometimes constant, whereas it sometimes varies very considerably as a function of the incomes considered in the two countries. This question will, it is hoped, be dealt with on another occasion.

TABLE I. — ANALYSIS OF VARIOUS VALUES OF D OBTAINED IN THE COMPARISON OF PRICES IN STOCKHOLM AND HELSINKI FOR DIFFERENT INCOME LEVELS

Stockholm 1925 (Stockholm prices for July 1930). Annual income per man-unit of consumption	Variables whose values are given in cols. 3-5	Finland 1920-1921 (Helsinki prices for July 1930): Annual income per man-unit of consumption		
		Mk. 4,500	Mk. 6,300	Mk. 14,900
(1)	(2)	(3)	(4)	(5)
Kr. 1,140	D	— 6.73 per cent.	— 7.39 per cent.	— 7.17 per cent.
	r	— 0.1896	— 0.2809	— 0.3778
	$\frac{\sigma_p}{P_0}$	$\frac{3.0933}{8.7184} = 0.3548$	$\frac{3.0933}{8.7184} = 0.3548$	$\frac{3.0933}{8.7184} = 0.3548$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.3491}{0.3487} = 1.0010$	$\frac{0.3072}{0.4142} = 0.7416$	$\frac{0.2358}{0.4407} = 0.5351$
Kr. 1,570	D	— 6.57 per cent.	— 7.23 per cent.	— 7.01 per cent.
	r	— 0.1862	— 0.2779	— 0.3751
	$\frac{\sigma_p}{P_0}$	$\frac{3.0945}{8.7033} = 0.3556$	$\frac{3.0945}{8.7033} = 0.3556$	$\frac{3.0945}{8.7033} = 0.3556$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.3034}{0.3058} = 0.9927$	$\frac{0.2656}{0.3630} = 0.7317$	$\frac{0.2030}{0.3862} = 0.5258$
Kr. 2,250	D	— 6.41 per cent.	— 7.07 per cent.	— 6.85 per cent.
	r	— 0.1772	— 0.2660	— 0.3671
	$\frac{\sigma_p}{P_0}$	$\frac{3.0743}{8.6882} = 0.3538$	$\frac{3.0743}{8.6882} = 0.3538$	$\frac{3.0743}{8.6882} = 0.3538$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.2710}{0.2651} = 1.0220$	$\frac{0.2365}{0.3149} = 0.7511$	$\frac{0.1767}{0.3350} = 0.5273$

TABLE II. — ANALYSIS OF VARIOUS VALUES OF D OBTAINED IN THE COMPARISON OF PRICES OF BERLIN AND WARSAW FOR DIFFERENT INCOME LEVELS

Germany 1927-28 (Berlin prices for July 1930). Annual income per man-unit of consumption	Variables whose values are given in cols. 3-5	Warsaw 1927 (Warsaw prices for July 1930): Annual income per man-unit of consumption		
		Zl. 500	Zl. 835	Zl. 1,540
(1)	(2)	(3)	(4)	(5)
RM. 700	D	—5.26 per cent.	—3.33 per cent.	—0.54 per cent.
	r	—0.2233	—0.1737	—0.0255
	$\frac{\sigma_p}{P_0}$	$\frac{0.4163}{1.4215} = 0.2929$	$\frac{0.4163}{1.4215} = 0.2929$	$\frac{0.4163}{1.4215} = 0.2929$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.6761}{0.8399} = 0.8049$	$\frac{0.7885}{1.2056} = 0.6541$	$\frac{1.2233}{1.7013} = 0.7190$
RM. 1,200	D	—4.08 per cent.	—2.12 per cent.	—0.71 per cent.
	r	—0.1540	—0.1093	—0.0382
	$\frac{\sigma_p}{P_0}$	$\frac{0.3889}{1.4039} = 0.2770$	$\frac{0.3889}{1.4039} = 0.2770$	$\frac{0.3889}{1.4039} = 0.2770$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.6225}{0.6511} = 0.9561$	$\frac{0.6534}{0.9345} = 0.6993$	$\frac{0.8805}{1.3187} = 0.6677$
RM. 3,400	D	—3.15 per cent.	—1.17 per cent.	—1.68 per cent.
	r	—0.0945	—0.0471	—0.0795
	$\frac{\sigma_p}{P_0}$	$\frac{0.3541}{1.3904} = 0.2547$	$\frac{0.3541}{1.3904} = 0.2547$	$\frac{0.3541}{1.3904} = 0.2547$
	$\frac{\sigma_q}{Q_0}$	$\frac{0.6856}{0.5239} = 1.3087$	$\frac{0.7325}{0.7519} = 0.9741$	$\frac{0.8830}{1.0611} = 0.8322$

cases presented in each table. Only one price series p_0 or p_1 was used for each country[1].

The first thing to note is that since the prices, p_0 and p_1, are the same for all the comparisons, $\frac{\sigma_p}{P_0}$ remains constant whenever P_0 is the same, that is to say, when different income classes in country 1 are compared with a particular income class in country 0. Further, $\frac{\sigma_p}{P_0}$ naturally varies when a change is made from one income class in country 0 to another, since P_0 is more or less affected. Provided that P_0 does not vary much, the variations in $\frac{\sigma_p}{P_0}$ will be relatively small.

For $\frac{\sigma_q}{Q_0}$ the variations will be fairly considerable if different income classes in country 1 are compared in turn with a particular income class in country 0.

If P_0 varies but little, it will be found that the variations in r are the inverse of the variations in $\frac{\sigma_q}{Q_0}$: when the dissimilarity of the quantities is small, the correlation r is fairly high and vice-versa.

In spite of the small number of cases presented, it is fairly clear that the variations of the elements forming D are considerable even if D itself remains practically stable. The question thus arises of determining which cases may be considered to offer the best guarantee of good comparability. Clearly, the results of the preceding analysis must determine the choice.

In applying them first to the comparison Sweden-Finland, it will at once be seen that the differences between the nine values of D are small. On this basis, even if it were sufficient, there is thus no guide for choice. The choice of the most comparable combination will therefore have to be founded on the basis of the sizes of r, $\frac{\sigma_p}{P_0}$ and $\frac{\sigma_q}{Q_0}$ and the ratio between $\frac{\sigma_q}{Q_0}$ and $\frac{\sigma_p}{P_0}$. Now r is large and of practically the same value in every case involving on the Finnish side the highest income. Also, $\frac{\sigma_q}{Q_0}$ is lowest, with nearly identical values, in these three cases shown in column 5. $\frac{\sigma_p}{P_0}$ being practically the same in all the combinations, it may be said that the choice

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For original figures see Appendix III below, pp. 96-103.

between the cases presented in column 5 is almost a matter of indifference unless preference is given to the combination involving the average income on the Swedish side. It is in this case that $\frac{\sigma_q}{Q_0}$ is smallest, while $\frac{\sigma_p}{P_0}$ is largest, which brings the two dissimilarities as close as possible, r being only very slightly lower than in the case of the combination of the highest Finnish income with the lowest Swedish income. Therefore, the combination Kr. 1,570 in Sweden with Mk. 14,900 in Finland offers the most favourable conditions for the comparison of prices, so that $D = -7.01$ per cent. should be taken into consideration as characterising comparability between Stockholm and Helsinki[1].

In the Berlin-Warsaw comparison the most striking feature is the comparatively marked variability of D, which ranges from —0.54 to —5.26 per cent. It will be seen at once, however, that the minimum value of D is due in the first place to the almost complete absence of correlation in the combination German minimum with Polish maximum. This case should therefore be left out of account without further consideration, as also certain others which need not be enumerated. Further, it appears that the case best satisfying the conditions that r should not be too small, and that the difference between $\frac{\sigma_p}{P_0}$ and $\frac{\sigma_q}{Q_0}$ should be as small as possible is the combination German minimum with Polish average. This is the combination, therefore which offers the best guarantee of comparability, and $D = -3.33$ per cent. indicates the degree of comparability of prices between Berlin and Warsaw[1].

In comparing the cases selected with those rejected it will be found that the application of the conditions of good comparability has led to a selection of those combinations in which the dissimilarity of the quantities consumed is small, not to say the smallest of all those calculated for the two countries in question. This fact is of some significance.

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[1] It will probably be useful to recall that since the whole enquiry is directed towards discovering the *relative* advantages that certain comparisons may possess over others, all that it is possible to infer from the results presented here, is that the combinations of incomes giving for D the values of —7.01 (for the comparison Stockholm-Helsinki) and —3.33 (for the comparison Berlin-Warsaw), are those considered the most suitable for a price comparison between these pairs of towns. In other words, they are the combinations for which P_0 and P_1 will most probably describe the limits within which the " true " price ratios are comprised. But there is no criterion at all for deciding whether this probability is high or low. At best, by making the necessary calculation for all the other towns and making allowance for all possible combinations of income, we could find the towns for which the best combinations from the standpoint of income would be more trustworthy than any other. But no inference at all should be drawn as to the absolute degree of confidence to be placed in these comparisons.

Consider on the one hand the expression for dissimilarity:

$$\text{(viii)} \qquad \frac{\sigma_q}{Q_0} = \sqrt{\frac{\Sigma\left(\frac{q_1}{q_0} - Q_0\right)^2 p_0 q_0}{p_0 q_0}} \cdot \frac{1}{Q_0}$$

and on the other hand the expression:

$$\text{(ix)} \qquad \Delta = \frac{\Sigma\left|\frac{q_1}{q_0} - Q_0\right| p_0 q_0}{p_0 q_0} \cdot \frac{1}{Q_0} \cdot$$

The analogy between these two expressions is evident. They are the weighted averages of the same deviations, the system of weights being identical. The only difference is that in the first case the square root of the weighted average of the squared deviations is divided by Q_0, whereas in the second case the weighted average of the same deviations taken irrespective of sign is divided by Q_0.

In view of this analogy, it is highly probable that the movements of the two expressions are analogous when they are calculated successively for the same income combinations in two countries.

Now, expression (ix) has been the subject of fairly thorough study[1], dealing precisely with the various values obtained when it is calculated for a series of incomes in country 1 in relation to a fixed income in country 0. It has been found that these values generally reach a minimum for a certain income in country 1, and this minimum shifts when the fixed income of country 0 is changed for another and the calculations are repeated for the whole series of incomes of country 1 in relation to the new base. If the values of expression (ix) are systematically calculated for all the possible combinations between the two income series for two countries, a continuous " valley " will be formed by the surface which has as co-ordinates the incomes of country 0, the incomes of country 1, and the values of expression (ix). By a line of reasoning based on appropriate and plausible hypotheses it is possible to arrive at the conclusion that the two incomes whose comparison gives a minimum value for expression (ix) are those for which the difference in real purchasing power is smaller than it would have been between one of the two and any income whatever in the other country. In other terms, these two incomes are those which are closest to equivalence in the sense of equality of standards of living.

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[1] Cf. H. Staehle, " Ein Verfahren zur Ermittlung gleichwertiger Einkommen in verschiedenen Ländern ", in *Archiv für Sozialwissenschaft und Sozialpolitik*, June 1932, and (by the same author) " The Reaction of Consumers to Changes in Prices and Incomes. A Quantitative Study on Immigrants' Behaviour ", *Econometrica*, January 1934, pp. 168-175. Numerous other applications will be published in the near future.

In view of the analogy between the two expressions (viii) and (ix) and the probability that variations in the first are analogous to those in the second, the same reasoning may apply in both cases. Whether the value of (viii) or of (ix) reaches a minimum in the sense of the preceding considerations, it seems equally legitimate to consider that the incomes for which this minimum is observed are nearly equivalent.

It is true that the present calculations have not been carried far enough and that the scales of income considered are too short and insufficiently graduated to make it clear whether the selected cases are in fact those for which comparability is the best possible and which give absolute minima for the dissimilarity of the quantities consumed. It is very probable, however, that if the material available on the one hand and the means of calculation on the other had allowed of carrying the experiment further by the consideration of a larger number of incomes in both cases (Sweden-Finland and Germany-Poland), it would have been possible to note the coincidence of the combination offering best comparability and that giving a minimum for expression (viii)[1].

The importance of these observations is considerable:

(1) The application of the conditions of good comparability has led to the selection of cases in which the dissimilarity of the quantities consumed is found to be small as compared with its value in other cases. This means to say that the comparison, if it is to be good, must relate to incomes in the two countries which are as equivalent as possible.

(2) In view of the fact that the minima of the two expressions

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[1] In spite of the already remarked inadequacy of our observations, which, placing us at the mercy of accidental fluctuations in the data considered, prevent us from recording a definitely continuous " valley of equivalence ", we have felt bound to give the values of expressions (viii) and (ix) for the cases

(viii) and (ix) may shift when changing from one fixed income in the base country to another, it is very possible that good comparability (coinciding with equivalence of the standards of living) will be found at two distinct income levels. But the degree of comparability, while being the highest possible at each of these levels, may nevertheless be different. It follows that the choice of the cases of best comparability may lead to the establishment of regions of entirely different composition according as the comparisons relate to a higher or lower standard of living.

(3) To establish a really reliable system of regions it would be necessary to make very extensive calculations, each case being thoroughly analysed. The object of these calculations should be to discover, so to speak, an international level of incomes on which an income for each country or each town should be situated, and which would be nearly equivalent to the incomes of other countries also placed on this level. It is on this level that comparisons could be safely made and that in cases where the comparisons between

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comprised in tables I and II. They are shown side by side in the following table.

Stockholm 1923 (Stockholm price for July 1930). Annual income per unit of consumption	Finland 1920-1921 (Helsinki price for July 1930): Annual income per unit of consumption					
	Mk. 4,500		Mk. 6,300		Mk. 14,900	
	Values of expressions					
	(viii)	(ix)	(viii)	(ix)	(viii)	(ix)
Kr. 1,140.	1.0010	0.4956	0.7416	0.4545	0.5351	0.4533
Kr. 1,570.	0.9927	0.4840	0.7317	0.4435	0.5258	0.4370
Kr. 2,250.	1.0220	0.4940	0.7511	0.4491	0.5273	0.4355

Germany 1927-1928 (Berlin price for July 1930). Annual income per unit of consumption	Warsaw 1927 (Warsaw price for July 1930): Annual income per unit of consumption					
	Zl. 500		Zl. 835		Zl. 1,540	
	Values of expressions					
	(viii)	(ix)	(viii)	(ix)	(viii)	(ix)
RM. 700	0.8049	0.7009	0.6541	0.5022	0.7190	0.4932
RM. 1,200	0.9561	0.8527	0.6993	0.6364	0.6677	0.4955
RM. 3,400	1.3087	1.0057	0.9741	0.7828	0.8322	0.6327

In this table the movements of these two series of values are on the whole seen to be fairly parallel. Nevertheless, there are some divergences. Without indicating each separately, it may be noted that the lowest value of expression (ix) is found in the first table for the combination of maximum incomes in Sweden and Finland, whereas the lowest value of expression (viii) is found for the combination medium-Sweden and maximum-Finland. It is probable that if we had taken the incomes between the medium and maximum in Sweden, lower values still would have been obtained for the two expressions as compared with the maximum in Finland. It is also probable that in the second table the " valley " would appear by taking the incomes between the medium and maximum in Warsaw and between the minimum and medium in Germany. In any case, the two combinations selected as offering the highest degree of comparability approach the " valley of equivalence " as it might be delineated by new calculations.

certain countries were unsatisfactory, a grouping of regions of good comparability could be made.

D. — Summary and Conclusion of Theoretical Considerations. Solution of a Last Difficulty

At this point we may briefly review the general course of the discussion. Starting from the principle that an ideal cost-of-living index should express the ratio between equivalent nominal incomes, we have adopted the well-known theorem, stated in Appendix I, that the two branches of formula (i) may be considered, for persons of the same tastes, as representing the upper and lower limits of such an index. When identity of tastes can be assumed in these two situations, the comparison will be the better the more nearly the branches of the formula coincide. Now, while it is possible to assume identical tastes at two near dates in one and the same town, this is certainly impossible for two different countries. In this case, the difference between P_0 and P_1 no longer suffices as a criterion of comparability, or of the value of the comparison, and a fuller analysis is required. In undertaking it, not only for determining the comparability of two towns, but also for comparisons at different income levels in one and the same town, we concluded that, to be as good as circumstances allow, a comparison utilising formula (i) should apply to incomes as nearly as possible equivalent. In an ideal regional system, direct comparisons should not be undertaken except for towns with a particularly high degree of comparability for the real-income level selected in the system. In addition, if the data allow, there should be a special regional system for every income level considered; in this way two towns could be compared directly as to a first income level and indirectly as to a second.

It will be agreed that these conclusions, which are in keeping with the point of departure, do not form a vicious circle, but they nevertheless raise a serious problem: if the indices apply only to equivalent incomes, how can they serve for the determination of the relative purchasing power of incomes of which only the nominal amount is known ?

It should be noted that this problem only arises when the numerical value of formula (i) varies with the income level considered—a comparatively rare case. Nevertheless, the problem will remain so long as the reason for such variation is unknown[1].

Supposing that it is desired to compare the purchasing power at time t' of two incomes R_0 and R_1 in countries 0 and 1, and the known quantities are: (1) the food-price indices connecting these two countries $I_{01} = \frac{1}{I_{10}}$ for a series of income levels; (2) budget enquiry data giving the nominal amounts of these incomes at a given date t; (3) the national cost-of-living indices from the date of these enquiries t to the date t'. Then, from these last indices we could determine to what nominal income on date t the two incomes R_0 and R_1 approximately correspond. This would allow us to select from the indices I_{01} the one to be applied to income R_0 to find the income R_1' equivalent to it at time t' in country 1; further, we could select from the indices I_{10} the one which, applied to income R_1, would give the income R_0' equivalent to it at time t' in country 0. We would then have two formulas for the relative purchasing power of incomes R_0 and R_1: the first $\frac{R_1'}{R_0}$, the second $\frac{R_1}{R_0'}$. But even if they were almost identical it would be inadvisable to reduce them to any sort of average. Further, if $\frac{R_1'}{R_0}$ gives the result 2,2 and $\frac{R_1}{R'_0}$ 2,0, it would be incorrect to say, for example, that income R_1 can yield about twice the satisfaction obtainable from income R_0. All that could be inferred is that to live at the same level in country 0 as with income R_1 in country 1, one would require a little more than twice the income R_0; and that an income of about half R_1 suffices to secure the same standard of living in country 1 as with income R_0 in country 0. This last difficulty would thus seem to be circumvented.

Turning now from the purely theoretical aspect, we shall seek the solution which, having regard to the limited means at our disposal, approximates as closely as possible to the conclusions reached in the first part of this memorandum[2].

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[1] See note on page 40. It will be observed that on this point we have modified the conclusions of the " Contribution " (see theorem (2), page 2).

[2] Comments on the relations between the above conclusions and the theories of other authors will be found in Appendix I, pp. 89-90 below.

III

FORMATION OF REGIONS OF GOOD COMPARABILITY

PRELIMINARY QUESTIONS OF A PRACTICAL NATURE

(a) *Practical Solution*

There is no need to say that in view of the limited means at the disposal of the Office it is impossible to apply completely the conclusions of the preceding analysis. In practice, therefore, all the refinements that might be indispensable from the theoretical point of view will have to be given up [1].

It will therefore have to be considered sufficient to apply criterion D without any means of submitting its values to that critical analysis which alone would allow of judging whether, while being small, they are significant from the point of view of good comparability. It may be noted, however, that the smaller the variations of D when the series q_0 and q_1 used relate to different incomes, the less serious will any objections to this procedure be. When D is altogether stable there is no problem of practical importance as to the choice to be made between the different series of weights. It may happen, however, even in this case that the small values obtained for D for all the possible combinations of income in the two countries in question would be due to values of r so small that it must be concluded that comparability between the two countries is bad in every case.

It has not always been possible, however, to determine whether D is relatively stable in this sense, either because the consumption of food is known only for the general averages of the families covered by the budget enquiry or because the means of calculation are lacking.

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[1] It is uncertain whether, even were it possible to overcome the difficulties of calculation, the available material would not be insufficient in the sense that the scales of income cover entirely different scales of standard of living in different countries. But then perhaps it would be possible to group countries into regions each having a different level of income.

In practice, it will therefore be considered sufficient to give, in addition to certain more detailed examples, the values of D when the calculation is based solely on general average consumption without further analysis. The comparison between countries 0 and 1 will be considered better than that between 0 and 2 if D is smaller in the first case.

(b) *Choice of the Countries observed and the Date of Comparison*

Before applying the method certain details and difficulties of a more or less practical nature should be noted.

Concerning first the scope of the experiment and the choice of countries, it has been found necessary, for this first enquiry, to keep to a few countries for which relatively recent budget enquiries are available. The scope of the observations has thus provisionally been confined to the following countries and towns[1]:

Austria (Vienna), Belgium (Brussels), Bulgaria (12 towns), Czechoslovakia (Prague), Denmark (Copenhagen), Estonia (Tallinn), Finland (Helsinki), Germany (Berlin), Latvia (Riga), Norway (Oslo), Poland (Warsaw), Sweden (Stockholm), Switzerland (34 communes).

For the date to which the experiment for the formation of regions relate, the choice has fallen on July 1930. Not only is fairly reliable information on retail prices available for this date as collected by the Office by means of special questionnaires, but it is also perhaps the most recent period in which the heavy fall in prices had not yet become too marked. It is clear that during a very rapid fall, such as that which took place in many countries in 1931 and 1932, it would have been necessary to take into consideration price averages covering longer periods so as to prevent perhaps incidental and passing differences in the rates of variation of the different prices from distorting the degrees of comparability between the various countries.

(c) *Data Used*

The choice of data to be used offers some difficulty owing to the fact that the dates to which the data of food consumption relate nowhere coincide with the date for which D is to be determined. It is evident that the quantity of each article consumed is a function of its price among other things, and in particular of its relative price (in this case this term denotes the ratio of a particular price to all other prices in the same locality). A modification of the latter will therefore necessarily involve a modification of the former. Strictly speaking, it is therefore inadmissible to assume that the quantities consumed will remain the same whatever may be the variations in prices and relative prices, as is done, however, whenever use is made of a series of q_0 or q_1 which is not observed simultaneously with the prices p_0 and p_1. It can be shown, however, that the result of formula (i) is not seriously affected by this kind of error. But this is not true for the values of P_0 and P_1.[1] Hence, it is necessary to make use as far as possible only of the most recent data of quantities consumed, this being the only precaution that can be taken in practice as regards the quantities.

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[1] For a more detailed survey of the material used see Appendix II, pp. 93-95.

For the prices p_0 and p_1, the data are taken from statistics of retail prices which are quite independent of the budget enquiries giving the quantity series q_0 and q_1. There are even fairly substantial divergences between the prices of the same articles actually paid during the period of the budget enquiry and those found for this period in the current statistics of retail prices. There is no doubt that these divergences are due to differences of quality in the sense that the price statistics always relate to a particular quality of each foodstuff, whereas the prices actually paid are weighted averages of the prices relating to different qualities (and if the period of the enquiry is long enough, to different dates). It therefore seems desirable to eliminate so far as possible this type of "error" in the price data serving for the calculation of the criterion of comparability. The correction is made by reducing or increasing the available prices for the period to which the calculations in question refer in proportion to the divergences existing at the time of the budget enquiry between the price recorded in current statistics and the price actually paid whenever this divergence exceeds 10 per cent. one way or the other. If, for instance, the price paid for a quantity q_0 of white bread in 1925 is 20 per cent. lower than the average price given for 1925 in the statistics of retail prices, the retail price of white bread for July 1930

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[1] "Contribution", pp. 222-224 and in particular tables II (p. 224) and IV (p. 227).

(the date for the calculation of D) is reduced in the same proportion [1].

The problem of the correspondence that ought to exist between the quantities consumed and prices cannot, however, be solved on practical grounds alone. The question arises in a general way whether the prices should relate to the same quality of each article in all the countries, or whether it is preferable that the quality to which the prices relate should be that for which consumption data are available. This question has previously been dealt with in another connection [2] and the reply then given calls for some amplification.

Assume first that the quality of each foodstuff for which the price is known for two towns is strictly identical. In this case there is a possibility that the prices may not in fact be the current prices of the quality usually consumed in either town; and the ratio between them, $\frac{p_1}{p_0}$, will not be identical with the ratio between the prices actually paid as a rule unless the ratio between the known price and the price of the quality most consumed is the same in both towns. If the price index is not to be affected, this ratio must in addition be the same for all articles. There is little chance that these conditions will be fulfilled, but unless they are, the reasoning set forth above, which assumed that $p_0 q_0$ and $p_1 q_1$ represent the actual expenditures of the inhabitants of the two localities, will not apply.

Further, assuming that p_0 and q_0 on the one hand, and p_1 and q_1 on the other relate to the same quality, this will at least guarantee that except for a possible modification of relative prices in the same locality between the date of the budget enquiry and the date chosen for the comparison, $p_0 q_0$ and $p_1 q_1$ will represent actual expenditures. As regards $p_1 q_0$ and $p_0 q_1$, these always represent fictitious expenditures. Where p_1 relates to quality q_1 and p_0 to quality q_0, these products represent the expenditure of an individual in country 0 (respectively country 1) who, transferred to country 1 (respectively 0), continues to consume his customary quantities but adapts himself to the new country in so far as he consumes for each article that quality which is most usual. This solution is not very satisfactory either. But though unlikely, the assumption on which it is based is certainly more reasonable than that resulting from making use only of prices relating to uniform qualities for all the countries, regardless of whether or not these qualities are consumed.

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[1] A description of certain other modifications applied to the original data is given in the introductory note to Appendix III, pp. 96-98.

[2] " Contribution ", p. 226.

To avoid any artificial hypothesis as to the adaptation of individuals, it would be necessary to know for each article considered in each town the price relating to the qualities ordinarily consumed in each of the other towns. Thus, for establishing P_0 only the current qualities in country 0 would be considered (each p_1 relating to the same quality as p_0 and q_0), and for establishing P_1 only the qualities current in country 1 (each p_0 relating to the same quality as p_1 and q_1). It will be agreed, however, that this refinement would introduce very considerable practical complications. It will therefore be sufficient here to ascertain that p_0 and q_0 on the one hand and p_1 and q_1 on the other relate to as similar qualities as possible.

(d) *Monetary Units*

The prices so corrected in order to eliminate at least part of the effect of the qualities of the articles considered are all known in units of national currency, and these prices, taken as they stand, have been used for the calculations. For this reason the first result of the price comparison expresses not only the difference between the price levels but also the relation between the monetary units. As in July 1930 none of the countries considered had yet abandoned the gold standard, it seems legitimate to eliminate this last influence by taking into account the relations between gold parities in order to bring out more clearly the differences between the price levels. Moreover, as the final object of the indexes constructed is the international comparison of the purchasing power of incomes expressed in national currencies, it is obvious that the question of the monetary unit is of no importance. It is only for reasons of convenience that the relation between gold parities has been eliminated from the results of the comparisons [1].

STRUCTURE OF THE REGIONS

The ground has now been sufficiently cleared from both the theoretical and the practical point of view to allow of applying the various conclusions reached.

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[1] In this connection cf. *International Labour Review*. Vol. XXVII, No. 4, April 1933, pp. 533-534.

It should be mentioned, however, that in spite of every precaution the regions cannot be formed without introducing a certain degree of arbitrariness. Even assuming that it has been possible to make an ideal and complete classification of the different cases according to their comparability, this will still not be sufficient to determine the regions in practice. For a final decision it becomes necessary to fix arbitrarily either the number of regions it is proposed to admit, or the conditions that any direct comparison must satisfy to be regarded as admissible.

Now the calculations made are far from complete. Not only has it been necessary to impose very severe limits upon the geographical scope of our experiments, but also to give up any detailed analysis of all the possible combinations between each town and all the other towns; and even the incomplete analysis does not cover each of these combinations. With a view to reducing the work of calculation, not even a single value for D has been calculated in certain cases in which it might be expected to be relatively large. Here again is a new arbitrary element in the result of the experiments.

Further, it has been considered advisable as a means not only of reducing the work involved, but in particular of facilitating the establishment and use of the indexes to be published periodically in future, to simplify the structure of the regions to some extent. Thus the comparisons are not made directly between all the localities in any one region, but the direct comparisons are made solely between each town in the region and a principal town—the " centre " —chosen in each region.

Each region is therefore composed of a nucleus, the centre, around which are grouped those of its towns for which direct comparisons with the centre are good. If a region comprises n localities, including the centre, all that is done is to establish n-1 indexes expressing the price ratio between each secondary locality and the centre. To compare any locality in the region with another it will thus be sufficient to determine the ratio between these two indexes. Hence, it is desirable that the indexes should satisfy the circular test.

Comparisons between towns belonging to different regions are made as follows: let I_A be an index of prices expressing the relation between a town in region A and its centre (the centre being taken as base), e.g. 0.60 or 60 per cent., and I_B a similar index for a town in region B, e.g. 0.80 or 80 per cent. Let the ratio of prices between the two centres of regions A and B be I_{AB}

(A being taken as base), e.g. 1.50 or 150 per cent. This last ratio will indicate the difference of level between the bases of all the known indexes for regions A and B. To compare the two towns to which index I_A and I_B relate it will be sufficient to take into account this difference of level expressed by I_{AB}. This can be done either by increasing $I_B = 80$ per cent. in the ratio 100 to 150, or by decreasing $I_A = 60$ per cent. in the ratio 150 to 100. The price ratio between these two towns will be in the first case 60 to 120 (base, centre of region A) and in the second 40 to 80 (base, centre of region B). The ratio is the same in both cases. In a general way it is expressed by $\frac{I_A}{I_{AB} \cdot I_B}$. It will be seen that once the regions have been determined, it will be enough to know the indexes of type I_A and I_B and those of type I_{AB} to compare the level of prices between any towns included in the system.

Assuming that there are six regions, each composed of ten towns, there will be for each region nine indexes of the type I_A and I_B, or in all 54. In addition, supposing that the formula used satisfies the condition of reversibility of base, a condition which is essential for interlocal comparisons, there will be 15 indexes of the type I_{AB}[1]. There would thus be 69 indexes to establish, which would be sufficient to allow of all the comparisons desired between all the 60 towns.

This procedure has therefore several advantages:

(1) The work of calculation involved in the current comparisons contemplated for the future will be substantially reduced.

(2) For towns belonging to the same region there will be indexes relating to the same base (prices at the centre) and directly comparable between themselves, provided that the comparison satisfies the circular test.

(3) Comparisons between towns belonging to different regions will be possible by means of the application of a fixed coefficient expressing the difference of level between the prices at the two centres.

These advantages are accompanied by a certain increase in the purely arbitrary elements of the procedure, in particular as regards the choice of centres, for this choice cannot be made without a simultaneous consideration of all the relations of comparability between all the localities considered. To begin with, however, a starting point must be chosen which, as will appear below, will not be taken at random (this being a possible procedure), but on the basis of previous experience. Notwithstanding the greatest precautions, consisting mainly in constant revision of the results provisionally acquired, the choice of this starting point may prejudice the results ultimately reached. Furthermore, while it is important that comparability should be good between towns in the same region, it is equally important that comparisons between the centres should be reliable, for it is clear that the badness of the comparison between two centres will affect all the comparisons between towns belonging to the two regions, whereas the badness of the comparison between any town and its centre can affect only the combined indexes in which this particular town is involved. It will be found that different systems of regions can be obtained according to the degree of importance attached to the comparability between centres on the one hand, and between towns in the same region (more precisely, particular localities and their respective centres) on the other. Finally, it is obvious that the results will also depend on what is considered good comparability and on the number of regions admitted. Taken as a whole, however, it would seem that the advantages enumerated above are considerably more important than the introduction of these few arbitrary elements.

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[1] It would even be possible to contemplate the selection of a " central centre " which would reduce the number of indexes of type I_{AB} from 15 to 5.

Choice of the First Centre: Stockholm or Oslo

The starting point, too, will be based on one of the results of the enquiry already mentioned [1]. This showed, as the result of calculations which were not published, that the comparisons of Finland with other countries gave values for P_0 and P_1 which are wide apart in every case with only one exception, namely in relation to Sweden and in particular Stockholm.

This experiment has been completely reaffirmed not only as regards the good comparability between Helsinki and Stockholm but also as regards the smaller comparability between Helsinki and certain other towns. This appears from table III, which shows,

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[1] " Contribution ".

TABLE III. — COMPARISON OF PRICES IN HELSINKI WITH THOSE IN STOCKHOLM, BERLIN, WARSAW AND OSLO IN JULY 1930

Reference No.	Number of foodstuffs considered	Total expenditure per unit in the year indicated of families whose consumption per man-unit = q_0		$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure in 1920-1921 of families in Finland whose consumption per man-unit = q_1 (in Finnish marks)	$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
		Town or country and year	Total expenditure per unit							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	20	Stockholm, 1923 .	Kr. 1,140	Kr. 215.82	Kr. 263.40	81.94	M. 4,500	Kr. 231.64	Kr. 303.12	76.42
2			,, 1,570	,, 245.82	,, 300.54	81.79	M. 6,300	,, 273.18	,, 360.02	75.88
3			,, 2,250	,, 282.87	,, 346.43	81.65	M. 14,900	,, 291.32	,, 383.02	76.06
4	19	Germany, 1927-1928 . . .	RM. 700	RM. 189.54	RM. 213.70	88.69	M. 4,500	RM. 226.06	RM. 319.60	70.73
5			,, 1,200	,, 221.49	,, 259.22	85.44	M. 6,300	,, 278.88	,, 396.55	70.33
6			,, 3,400	,, 254.57	,, 308.33	82.56	M. 14,900	,, 315.26	,, 443.88	71.02
7	16	Warsaw, 1927. . .	Zl. 2,200	Zl. 333.58	Zl. 230.93	144.45	M. 4,500	Zl. 451.40	Zl. 417.58	108.10
8			,, 3,100	,, 449.35	,, 328.21	136.90	M. 6,300	,, 558.06	,, 518.87	107.55
9			,, 4,100	,, 620.74	,, 475.15	130.64	M. 14,900	,, 613.60	,, 556.46	110.27
10	18	Norway, 1927-1928	Kr. 4,000	Kr. 232.84	Kr. 290.80	80.07	M. 4,500	Kr. 198.34	Kr. 291.31	68.09
11			,, 4,500	,, 252.35	,, 320.34	78.78	M. 6,300	,, 245.59	,, 361.19	67.99
12			,, 5,600	,, 288.82	,, 363.52	79.45	M. 14,900	,, 277.01	,, 408.70	67.78

on the one hand the values of P_0 and P_1 for the comparison Stockholm-Helsinki for which a complete analysis was given above, and on the other hand the values of P_0 and P_1 *resulting from the comparisons Berlin-Helsinki,* Warsaw-Helsinki and Oslo-Helsinki *for various income combinations. For the choice of income* use has generally been made of the practice already followed above of taking in each country the lowest, an intermediate (sometimes the general average) and the highest income groups for which the average quantities consumed are given in budget enquiries. In this way three different values are obtained for P_0 and P_1, according as the q_0's or q_1's represent the quantities consumed relating to these three income classes. It has been thought advisable to give in addition to these various values of P_0 and P_1 the sums of which they are the ratio, since in themselves these form a very instructive subject for study [1]; but while this aspect of the results is presented, it is not discussed in detail. As regards D, nine different values can be calculated in each case according to the various combinations possible between the three values of P_0 and the three values of P_1. It will be sufficient in general to cite in the text the extreme values that D can have, without presenting them all. If need be, they are easy to calculate.

Table III shows that D is relatively small for the comparison Stockholm-Helsinki, but that on the other hand it is fairly substantial in the cases Berlin-Helsinki, Warsaw-Helsinki and Oslo-Helsinki. Its variations are small for Stockholm-Helsinki (between — 6.41 and — 7.39 per cent.) and Oslo-Helsinki (between — 13.57 and — 15.35 per cent.), but very considerable for Berlin-Helsinki (between — 13.98 and — 20.25 per cent.), and for Warsaw-Helsinki (between — 15.59 and — 25.55 per cent.). It is therefore easy to conclude that Helsinki will always be relatively comparable with Stockholm but not at all comparable with Berlin and Warsaw. For Oslo there is not sufficient material on which to base a definite rejection or acceptance. Only by an analysis of the kind given in tables I and II would it be possible to take a final decision on the subject. It being impossible to arrive at a complete judgment of the respective merits of Stockholm and Oslo in the comparison with

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[1] It is hoped that these data will be used in the near future in order to enquire into the question already raised (see note on p. 30), why D is in some cases almost indifferent to, whereas in other cases it reacts very strongly upon, the variations of the incomes considered in two countries.

TABLE IV. — COMPARISON OF PRICES IN STOCKHOLM WITH THOSE IN FIVE OTHER TOWNS IN JULY 1930

Reference No.	No. of foodstuffs considered	Total annual expenditure per man-unit in 1923 of Stockholm families whose consumption $= q_0$	$\Sigma\, p_1 q_0$	$\Sigma\, p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose consumption per man-unit $= q_1$		$\Sigma\, p_1 q_1$	$\Sigma\, p_0 q_1$	$P_1 \times 100$
						Town or country	Expenditure			
16	22	Kr. 1,140	Kr. 287.69	Kr. 266.70	107.87	Oslo, 1927-1928	Kr. 1,175	Kr. 299.13	Kr. 298.44	100.23
17		,, 1,570	,, 331.12	,, 306.01	108.21		,, 1,430	,, 329.59	,, 323.97	101.73
18		,, 2,250	,, 385.29	,, 355.31	108.44		,, 2,450	,, 375.72	,, 363.20	103.45
19	20	Kr. 1,140	Kr. 263.96	Kr. 263.65	100.12	Denmark, 1922 (Copenhagen prices)	Kr. 1,600 . under	Kr. 222.55	Kr. 252.66	88.08
20		,, 1,570	,, 304.62	,, 302.12	100.83		,, 2,400	,, 265.81	,, 295.56	89.93
21		,, 2,250	,, 353.60	,, 348.96	101.33		,, 3,200 and over	,, 288.30	,, 314.59	91.64
22	23	Kr. 1,140	RM. 330.03	RM. 304.74	108.30	Germany, 1927-1928 (Berlin prices)	RM. 700	RM. 217.98	RM. 247.26	88.16
23		,, 1,570	,, 378.61	,, 348.96	108.50		,, 1,200	,, 266.29	,, 286.86	92.83
24		,, 2,250	,, 439.08	,, 403.75	108.75		,, 3,400	,, 324.40	,, 330.15	98.26
25	19	Kr. 1,140	Zl. 429.55	Zl. 585.99	73.30	Warsaw, 1927	Zl. 2,200	Zl. 234.88	Zl. 474.05	49.55
26		,, 1,570	,, 491.96	,, 669.57	73.47		,, 3,100	,, 341.47	,, 642.52	53.15
27		,, 2,250	,, 567.89	,, 773.35	73.43		,, 4,100	,, 496.03	,, 862.84	57.49
28	17	Kr. 1,140	Kc. 2,106.17	Kc. 2,197.87	95.83	Czechoslovakia, 1927-1928 (Prague prices)	Kc. 5,820	Kc. 2,083.25	Kc. 2,455.59	84.84
29		,, 1,570	,, 2,421.08	,, 2,507.65	96.55		,, 12,300	,, 2,287.01	,, 2,556.75	86.08
30		,, 2,250	,, 2,796.30	,, 2,878.24	97.15		,, 20,380	,, 2,360.38	,, 2,707.94	87.16

TABLE V. — COMPARISON OF PRICES IN OSLO WITH THOSE IN FIVE OTHER TOWNS IN JULY 1930

Reference No.	No. of foodstuffs considered	Total annual expenditure per man-unit in 1927 of Oslo families whose consumption = q_0	$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose consumption per man-unit = q_1		$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
						Town or country	Expenditure			
31	22	Kr. 1,175	Kr. 298.44	Kr. 299.13	99.77	Stockholm, 1923	Kr. 1,140	Kr. 266.70	Kr. 287.69	92.70
32		,, 1,430	,, 323.97	,, 329.59	98.29		,, 1,570	,, 306.01	,, 331.12	92.42
33		,, 2,450	,, 363.20	,, 375.72	96.67		,, 2,250	,, 355.31	,, 385.29	92.22
34	19	Kr. 1,175	Kr. 251.98	Kr. 289.95	86.90	Denmark, 1922 (Copenhagen prices)	Kr. 1,600 a. under	Kr. 215.94	Kr. 254.52	84.84
35		,, 1,430	,, 279.00	,, 318.75	87.53		,, 2,400	,, 259.63	,, 301.09	86.23
36		,, 2,450	,, 318.03	,, 364.83	87.17		,, 3,200 and over	,, 284.58	,, 327.14	86.99
37	22	Kr. 1,175	RM. 331.10	RM. 336.52	98.39	Germany, 1927-1928 (Berlin prices)	RM. 700	RM. 211.50	RM. 228.36	92.62
38		,, 1,430	,, 362.64	,, 370.79	97.80		,, 1,200	,, 259.95	,, 278.71	93.27
39		,, 2,450	,, 404.73	,, 422.69	95.75		,, 3,400	,, 320.60	,, 341.39	93.91
40	19	Kr. 1,175	Kr. 186.58	Kr. 272.00	68.60	Warsaw, 1927	Zl. 2,200	Kr. 98.04	Kr. 170.32	57.56
41		,, 1,430	,, 203.33	,, 300.82	67.59		,, 3,100	,, 142.94	,, 243.07	58.81
42		,, 2,450	,, 232.11	,, 346.35	67.02		,, 4,100	,, 207.64	,, 349.08	59.48
43	16	Kr. 1,175	Kr. 227.68	Kr. 253.29	89.89	Czechoslovakia, 1927-1928 (Prague prices)	Kc. 5,820	Kr. 230.32	Kr. 287.56	80.09
44		,, 1,430	,, 249.98	,, 280.51	89.12		,, 6,540	,, 231.43	,, 289.48	79.95
45		,, 2,450	,, 282.78	,, 316.36	89.39		,, 20,370	,, 260.96	,, 320.98	81.30

Finland, the dilemma can be solved only by what subsequent results may be attained [1].

For this reason Stockholm and Oslo are selected as candidates for the centre of a first region.

Composition of the First Region

Consider first the comparability between Stockholm and other towns; table IV gives an analysis, relating to the comparison based not only on the general averages of the quantities consumed, but of the quantities for three income classes on each side.

It will be seen that the values of D are relatively small for the comparisons of Stockholm with Norway (Oslo), Denmark (Copenhagen) and Czechoslovakia (Prague); on the other hand, they are larger for Stockholm-Germany (Berlin) and Stockholm-Warsaw. As regards their variability, it is relatively small for Oslo, Copenhagen and Prague, and relatively large for Berlin and Warsaw.

It thus appears that the comparisons Stockholm-Oslo, Stockholm-Copenhagen and Stockholm-Prague may be described as relatively good, while the cases Stockholm-Berlin and Stockholm-Warsaw may be rejected as bad.

Next, the comparability between Oslo and other towns may be considered by way of analyses of the same kind, given in table V.

The table shows that the Oslo prices are comparable in all the cases considered, when this judgment is based on the same criteria as for the Stockholm prices. Though D is sometimes large, its variations are comparatively small.

Table VI, brings together the extreme values of D that can be calculated from the data in tables IV and V. It follows that from the two points of view of the size and the variability of D, Oslo is in a much more favourable position than Stockholm except with respect to Helsinki.

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[1] Mention may also be made of the following values for the comparison, based on the general averages of quantities, of three other towns with Helsinki, taken as base:

Reference No.		P_1	P_0	D
13	Helsinki-Riga	93.12	76.36	— 18.00 per cent
14	Helsinki-Tallinn	79.16	65.61	— 17.12 „ „
15	Helsinki-Copenhagen. . .	152.12	108.92	— 28.40 „ „

To judge from the values of D, the comparability does not appear to be good.

TABLE VI. — SUMMARY OF THE ELEMENTS DETERMINING THE COMPARABILITY OF DIFFERENT TOWNS WITH STOCKHOLM AND OSLO.

Extreme values of *D* obtained in the comparisons between				
towns " 1 "	and towns " 0 "			
	Stockholm		Oslo	
	Minimum	Maximum	Minimum	Maximum
Stockholm . . .	—	—	— 4.10	— 7.57
Oslo	— 4.10	— 7.57	—	—
Copenhagen . . .	— 8.47	— 13.08	+ 1.04 [1]	— 3.07
Berlin	— 9.27	— 18.93	— 1.92	— 5.86
Warsaw	— 21.57	— 32.56	— 11.25	— 16.09
Prague	— 9.05	— 12.67	— 8.77	— 11.06
Helsinki	— 6.41	— 7.39	— 13.57	— 15.35

[1] The positive value in this case can be explained by the fact that the comparison relates to the lowest income on the Norwegian side and the highest on the Danish side. The fact that even the maximum value of *D* is very small shows that the situation in the two countries approaches identity; and it would be sufficient to calculate the values of r, $\frac{\sigma_p}{P_0}$ and $\frac{\sigma_q}{Q_0}$ to find that probably they would all be very small. In these conditions, r can be very small and positive or negative without necessarily indicating non-comparability (See later on pp. 55-58).

Two questions thus arise to which only an arbitrary answer is possible:

(1) Is it wished to have a distinct " Nordic region ", comprising only countries ordinarily considered as northern, or can a geographically heterogeneous composition be accepted for the first region ?

(2) Should greater importance be attached (*a*) to the comparability between towns belonging to any one region and their centre, or (*b*) to the comparability between centres ?

It will be seen that, subject to comparability relations still unknown, and the possible modifications involved, the results will be quite different, even in this first stage, according to the replies given to the above questions.

Assuming that a distinct " Nordic region " is desired, and that it is considered that comparability within the region is more important than comparability between centres, then we may choose Stockholm as the centre for the north, with Helsinki, Oslo and (provisionally) Copenhagen. At the same time, Prague might be considered as the centre of a new region [1].

Assuming on the other hand that while wishing to have a Nordic region, it is preferred to accept the comparison between Helsinki and Oslo as sufficiently good while attaching greater importance to the comparability between centres, then Oslo might serve as a centre for the North, with Helsinki, Stockholm and (provisionally) Copenhagen. In this case Berlin and Prague, and possibly even Warsaw, might be taken as candidates for centres. But if geographical considerations are left out of account, and good comparability within the region is considered more important, then Berlin and Prague might (provisionally) be attached to Oslo as centre.

It has been thought necessary to enter into these details in order to show how arbitrary the choice of regions must be. There is of course no intention of pursuing the detailed enumeration of all possible combinations; the object is solely to bring out the necessity of taking essentially arbitrary decisions. Furthermore, it is proposed to consider two distinct systems of regions, with a view to studying the effect on the final indexes to which they lead. Hence it is decided:

(1) that a Nordic region is desired [2];

(2) that for the centre of this region the town of Stockholm will be taken in the first system, and that of Oslo in the second.

These decisions mean that in the first system the town of Prague will be the centre of a central region, and in the second system Berlin, Prague or Warsaw. But before proceeding to construct these new regions, a final decision must be taken as to Copenhagen, which has only provisionally been placed in the northern system.

Tables VII and VIII provide the necessary elements for taking this decision. They show that the comparability of Copenhagen is excellent with Oslo, Warsaw, and Berlin, but less good with

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[1] The possibility is rejected here of including Prague in the region with Stockholm as centre, owing to the unsatisfactory comparability between Stockholm on the one hand and either Berlin or Warsaw on the other. It would hardly be possible to take either of these towns as a new centre, for the inter-regional comparisons would be too unreliable.

[2] This has the advantage of grouping countries of similar climate, thus facilitating a future comparison of heating and lighting costs in the same system of regions.

TABLE VII. — COMPARISON OF PRICES IN COPENHAGEN WITH THOSE IN 5 OTHER TOWNS IN JULY 1930

Reference No.	Number of foodstuffs considered	Total expenditure per unit in the year indicated of families whose consumption per man-unit = q_0		$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure in 1922 of families in Copenhagen whose consumption per man-unit = q_1	$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
		Town or country and year	Total expenditure per unit							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
46 47 48	20	Stockholm, 1923 .	Kr. 1,140 ,, 1,570 ,, 2,250	Kr. 263.96 ,, 304.62 ,, 353.60	Kr. 263.65 ,, 302.12 ,, 348.96	100.12 100.83 101.33	Kr. 1,600 and less ,, 2,400 ,, 3,200 and more	Kr. 222.55 ,, 265.81 ,, 288.30	Kr. 252.66 ,, 295.56 ,, 314.59	88.08 89.93 91.64
49 50 51	19	Oslo, 1927-1928. .	Kr. 1,175 ,, 1,430 ,, 2,450	Kr. 251.98 ,, 279.00 ,, 318.03	Kr. 289.95 ,, 318.75 ,, 364.83	86.90 87.53 87.17	,, 1,600 and less ,, 2,400 ,, 3,200 and more	Kr. 215.94 ,, 259.63 ,, 284.58	Kr. 254.52 ,, 301.09 ,, 327.14	84.84 86.23 86.99
52 53 54	22	Germany, 1927-1928 (Berlin prices) .	RM. 700 ,, 1,200 ,, 3,400	RM. 215.31 ,, 274.17 ,, 337.94	RM. 241.01 ,, 302.48 ,, 369.83	89.34 90.64 91.38	,, 1,600 and less ,, 2,400 ,, 3,200 and more	RM. 263,06 ,, 310.23 ,, 334.74	RM. 300,72 ,, 350.93 ,, 379.52	87.48 88.40 88.20
55 56 57	17	Czechoslovakia, 1927-1928 (Prague prices) .	Kc. 5,820 ,, 12,300 ,, 20,380	Kc. 2,320.68 ,, 2,587.41 ,, 2,726.62	Kc. 2,094.57 ,, 2,371.84 ,, 2,467.07	110.80 109.09 110.52	,, 1,600 and less ,, 2,400 ,, 2,200 and more	Kc. 1,912.47 ,, 2,260.44 ,, 2,428.58	Kc. 1,918.35 ,, 2,229.61 ,, 2,347.81	99.69 101.38 103.44
58 59 60	18	Warsaw, 1927 . .	Zl. 2,200 ,, 3,100 ,, 4,100	Zl. 317.84 ,, 469.01 ,, 688.19	Zl. 241.24 ,, 353.51 ,, 515.20	131.75 132.67 133.58	,, 1,600 and less ,, 2,400 ,, 3,200 and more	Zl. 463.21 ,, 575.91 ,, 635.85	Zl. 360.62 ,, 439.98 ,, 483.56	128.45 130.89 131.49

TABLE VIII. — COMPARABILITY OF COPENHAGEN WITH CERTAIN OTHER TOWNS

Extreme values of *D* obtained in the comparison between Copenhagen		
and	Minimum	Maximum
Stockholm	— 8.47	— 13.08
Oslo	+ 1.04	— 3.07
Berlin	— 1.05	— 4.27
Prague	— 5.18	— 10.03
Warsaw	— 0.20	— 3.84

Prague, and mediocre with Stockholm. From this it may be concluded that in system I Copenhagen will not belong to the Stockholm region, but should be placed with Prague taken as possible centre for another region; in system II Copenhagen may be associated with the Oslo region, but might just as well be placed with Warsaw, Berlin or even Prague. It is only for geographical and climatic reasons that Copenhagen will be placed with Oslo in system II.

The composition of the Nordic region will thus be as follows:

System I: Centre: Stockholm,
Other towns: Helsinki, Oslo.
System II: Centre: Oslo,
Other towns: Stockholm, Helsinki, Copenhagen.

COMPOSITION OF THE OTHER REGIONS

As the preceding considerations will have shown, the construction of regions in system I rests primarily on good comparability within each region, whereas in system II comparability must be good in particular between the centres. As prospective centres for system I we have been led to contemplate the towns of Copenhagen and Prague, and for system II in the first place Berlin and possibly Prague and Warsaw. The next point to consider, therefore, is the degree of comparability between these four towns and the other localities so as to constitute the other region or regions in the two systems.

Tables IX to XII set forth all the elements needed to give a first idea of the relative advantages of these four towns as centres. Table XIII gives the two extreme values of *D* for the cases in which the analysis relates to several income classes, and the value of D obtained on the basis of the general averages of the quantities consumed for the remaining cases.

Taking system I, we may consider the position of Copenhagen and Prague. As in this system the chief point is comparability within the region, it is hardly possible to contemplate more than Vienna and Switzerland for the Prague region, for comparability between Prague and Berlin cannot be considered excellent owing to the relatively large variability of D. As regards Warsaw, the variability is slightly less, but D is too large on the average for the town to be included with Prague in system I. Without going further in this direction, we may consider the position of Copenhagen, for which it will be remembered comparability with Stockholm is mediocre but sufficient to allow of taking Copenhagen as centre, side by side with the Stockholm region. It will be seen that Warsaw and Berlin, both of which compare more or less badly with Prague, compare very well with Copenhagen, and even Prague compares on the whole as well with Copenhagen as with Stockholm, if not better[1]. Precisely for this reason it would be useful to have two centres: Copenhagen, with Warsaw, Berlin and Brussels (although Brussels is rather more comparable with Warsaw); and Prague, with Switzerland and Vienna (although Vienna is rather more comparable with Berlin)[2]. For the two Baltic towns Riga and Tallinn, the first compares only with Warsaw, the second with Warsaw and (though not so well) with Copenhagen[3]. It therefore becomes indispensable to make Warsaw, if not an independent

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[1] It is seen here how useful it would be to find a criterion by which to choose between a slightly greater variability of D accompanied by a slightly smaller average value of D on the one hand and a slightly smaller variability accompanied by a rather larger average value of D. Such a criterion might lead to the inclusion of Oslo in the Copenhagen region in system I.

[2] Here the direct comparison Vienna-Copenhagen is left out of account, for which a positive value for D is obtained. In other cases, analysed rather more thoroughly, positive values were admitted (e.g. Copenhagen-Oslo, see in particular the note to table VI, p. 44), since there was reason to think that r, $\frac{\sigma p}{P_0}$ and $\frac{\sigma p}{Q_0}$ were all small and that the situation approached identity. Here there is only one value for D and others could not be calculated because the Austrian data are not classified according to income. Hence, the small positive value of D is considered an indication of non-comparability between Vienna and Copenhagen.

[3] In support of this affirmation the comparison of two Baltic towns with Stockholm may be given.

Ref. No.		P_0	P_0	D
130	Stockholm-Riga	76.06	57.70	— 24.14 %
131	Stockholm-Tallinn	67.02	47.77	— 28.72 %

Reference No.	Number of foodstuffs considered	Total annual expenditure per unit in 1927-1928 of German families whose annual consumption per man-unit = q_0	$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose annual consumption per man-unit = q_1		$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
						Country or town and year	Expenditure			
61	23	RM. 700	RM. 247.26	RM. 217.98	113.43	Stockholm, 1923	Kr. 1,140	RM. 304.74	RM. 330.03	92.34
62		,, 1,200	,, 286.86	,, 266.29	107.73		,, 1,570	,, 348.96	,, 378.61	92.17
63		,, 3,400	,, 330.15	,, 324.40	101.77		,, 2,250	,, 403.75	,, 439.08	91.95
64	22	,, 700	,, 228.36	,, 211.50	107.97	Oslo, 1927-1928	,, 1,175	,, 336.52	,, 331.10	101.64
65		,, 1,200	,, 278.71	,, 259.95	107.22		,, 1,430	,, 370.79	,, 362.64	102.25
66		,, 3,400	,, 341.79	,, 320.60	106.48		,, 2,450	,, 422.69	,, 404.73	104.44
67	20	,, 700	,, 215.31	,, 241.01	89.34	Denmark, 1922 (Copenhagen prices)	,, 1,600 a. under	,, 263.06	,, 300.72	87.48
68		,, 1,200	,, 274.17	,, 302.48	90.64		,, 2,400	,, 310.23	,, 350.93	88.40
69		,, 3,400	,, 337.94	,, 369.93	91.38		,, 3,200 and over	,, 334.74	,, 379.53	88.20
70	20	,, 700	,, 144.24	,, 215.47	66.94	Warsaw, 1927	Zl. 2,200	,, 114.77	,, 180.98	63.42
71		,, 1,200	,, 183.78	,, 277.98	66.11		,, 3,100	,, 168.10	,, 259.76	64.71
72		,, 3,400	,, 226.19	,, 345.45	65.48		,, 4,100	,, 244.06	,, 366.57	66.58
73	18	,, 700	,, 204.63	,, 228.92	89.39	Czechoslovakia, 1927-1928 (Prague prices)	Kc. 5,820	,, 262.35	,, 308.05	85.17
74		,, 1,200	,, 252.58	,, 285.40	88.50		,, 12,300	,, 297.52	,, 352.51	84.40
75		,, 3,400	,, 295.60	,, 342.29	86.36		,, 20,380	,, 310.54	,, 371.98	83.48
76	20	,, 1,200	,, 282.46	,, 258.83	109.13	Winterthur, 1922 (prices for 34 towns)	Frs. 2,025	,, 400.52	,, 392.58	102.02
77	25	,, 1,200	,, 187.28	,, 307.43	60.92	Tallinn, 1925	EKr. 400	,, 210.16	,, 393.70	53.38
78	23	,, 1,200	,, 197.42	,, 286.08	69.01	Riga, 1926-1927	—	,, 221.74	,, 358.18	61.91
79	22	,, 1,200	,, 302.39	,, 297.16	101.76	Vienna, 1930	Sch. 1,500 per pers.	,, 356.98	,, 367.61	97.11
80	15	,, 1,200	,, 203.69	,, 246.31	82.70	Belgium, 1928-1929 (Brussels prices)	Frs. 5,255	,, 310.07	,, 398.42	77.83
81	17	,, 1,200	,, 142.57	,, 229.91	62.01	Bulgaria, 1927-1928	Levas 14,850	,, 118.50	,, 245.49	48.27

TABLE X. — COMPARISON OF PRICES IN WARSAW WITH THOSE IN 11 OTHER TOWNS IN JULY 1930

Reference No.	Number of foodstuffs considered	Total annual expenditure per unit in 1927 of Warsaw families whose annual consumption per man-unit = q_0	$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose annual consumption per man-unit = q_1: Country or town and year	Total annual expenditure per unit in the year indicated of families whose annual consumption per man-unit = q_1: Expenditure	$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
82	19	Zl. 500	Zl. 474.05	Zl. 234.89	201.82	Stockholm, 1923	Kr. 1,140	Zl. 585.99	Zl. 429.55	136.42
83		,, 835	,, 642.52	,, 341.47	186.16		,, 1,570	,, 669.57	,, 491.96	136.10
84		,, 1,540	,, 862.84	,, 496.03	173.95		,, 2,250	,, 773.35	,, 567.89	136.18
85	19	,, 500	Kr. 170.32	Kr. 98.04	173.73	Oslo, 1927-1928	Kr. 1,175	Kr. 272.00	Kr. 186.58	145.78
86		,, 835	,, 243.07	,, 142.94	170.05		,, 1,430	,, 300.82	,, 203.33	147.95
87		,, 1,540	,, 349.09	,, 207.64	168.12		,, 2,450	,, 346.35	,, 232.11	149.22
88	18	,, 500	Zl. 317.84	Zl. 241.24	131.75	Denmark, 1922 (Copenhagen prices)	,, 1,600 a. under	Zl. 463.21	Zl. 360.62	128.45
89		,, 835	,, 469.01	,, 353.51	132.67		,, 2,400	,, 575.91	,, 439.98	130.89
90		,, 1,540	,, 688.19	,, 515.20	135.58		,, 3,200 and over	,, 635.85	,, 483.56	131.49
91	17	,, 500	,, 336.85	,, 239.35	140.74	Czechoslovakia, 1927-1928 (Prague prices)	Kc. 5,820	Zl. 545.99	,, 448.12	121.84
92		,, 835	,, 483.19	,, 351.10	137.62		,, 12,300	,, 624.21	,, 508.05	122.86
93		,, 1,540	,, 679.17	,, 505.23	134.43		,, 20,380	,, 648.74	,, 526.96	123.11
94	20	,, 500	,, 384.30	,, 243.71	157.69	Germany, 1927-1928 (Berlin prices)	RM. 700	Zl. 457.54	,, 306.28	149.39
95		,, 835	,, 551.59	,, 356.95	154.53		,, 1,200	,, 590.28	,, 390.25	151.26
96		,, 1,540	,, 778.40	,, 518.26	150.19		,, 3,400	,, 733.55	,, 480.31	152.72
97	23	,, 835	,, 308.98	,, 370.94	83.30	Tallinn, 1925	EKr. 400	,, 398.51	,, 480.41	82.95
98	23	,, 835	,, 352.27	,, 373.38	94.35	Riga, 1926-1927	—	,, 458.72	,, 492.12	93.21
99	19	Zl. 920 (1929)	,, 580.40	,, 366.29	158.45	Vienna, 1930	Sch. 1,500 per pers.	,, 749.11	,, 517.02	144.19
100	17	,, 920 (1929)	,, 286.71	,, 311.71	91.98	Bulgaria, 1927	Levas 14,850	,, 251.60	,, 325.97	77.19
101	19	,, 835	,, 623.83	,, 345.81	180.40	Winterthur, 1922	Frs. 2,025	,, 842.03	,, 533.16	161.68
102	14	,, 835	,, 412.53	,, 315.21	130.87	Belgium, 1928-1929	Frs. 5,225	,, 644.52	,, 509.22	126.57

Reference No.	Number of foodstuffs considered	Total annual expenditure per unit in 1927-1928 of Czechoslovak families whose annual consumption per man-unit = q_0	$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose annual consumption per man-unit = q_1		$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
						Country or town and year	Expenditure			
103	17	Kc. 5,820	Kc. 2,455.59	Kc. 2,083.25	117.87	Stockholm, 1923	Kr. 1,140	Kc. 2,197.87	Kc. 2,106.17	104.35
104		,, 12,300	,, 2,656.75	,, 2,287.01	116.17		,, 1,570	,, 2,507.65	,, 2,421.08	103.58
105		,, 20,380	,, 2,707.94	,, 2,360.38	114.72		,, 2,250	,, 2,878.24	,, 2,796.30	102.93
106	16	,, 5,820	,, 287.56	,, 230.32	124.85	Oslo, 1927-1928	,, 1,175	Kr. 253.29	Kr. 227.68	111.25
107		,, 6,540	,, 289.48	,, 231.43	125.08		,, 1,420	,, 280.51	,, 249.98	112.21
108		,, 20,380	,, 320.98	,, 260.96	123.00		,, 2,450	,, 316.36	,, 282.78	111.87
109	17	,, 5,820	Kc. 2,320.68	Kc. 2,094.57	110.80	Denmark, 1922 (Copenhagen prices)	,, 1,600 a. under	Kc. 1,912.47	Kc. 1,918.35	99.69
110		,, 12,300	,, 2,587.41	,, 2,371.28	109.09		,, 2,400	,, 2,260.44	,, 2,229.61	101.38
111		,, 20,380	,, 2,726.62	,, 2,467.07	110.52		,, 3,200 and over	,, 2,428.58	,, 2,347.81	103.44
112	18	,, 5,820	RM. 308.05	RM. 262.35	117.42	Germany, 1927-1928 (Berlin prices)	RM. 700	RM. 228.92	RM. 204.63	111.87
113		,, 12,300	,, 352.51	,, 297.52	118.48		,, 1,200	,, 285.40	,, 252.58	112.99
114		,, 20,380	,, 371.98	,, 310.54	119.78		,, 3,400	,, 342.29	,, 295.60	115.79
115	17	,, 5,820	Zl. 448.12	Zl. 545.99	82.07	Warsaw, 1927	Zl. 500	Zl. 239.35	Zl. 336.85	71.06
116		,, 12,300	,, 508.05	,, 624.21	81.39		,, 835	,, 351.10	,, 483.19	72.66
117		,, 20,380	,, 526.96	,, 648.74	81.23		,, 1,540	,, 505.23	,, 679.17	74.39
118	20	,, 12,300	Kc. 1,961.23	Kc. 2,469.58	79.42	Tallinn, 1925	EKr. 400	Kc. 1,449.91	Kc. 2,229.60	65.03
119	19	,, 12,300	,, 2,025.03	,, 2,441.02	82.96	Riga, 1927-1927	—	,, 1,656.10	,, 2,257.86	73.35
120	16	,, 12,300	,, 1,650.04	,, 2,244.03	73.53	Bulgaria, 1927-1928	Levas 14,850	,, 948.19	,, 1,494.44	63.45
121	15	,, 12,300	,, 2,331.94	,, 2,171.79	107.37	Belgium, 1928-1929	Frs. 5,225	,, 2,492.99	,, 2,705.61	92.14
122	19	,, 6,540	Sch. 569.35	Sch. 461.43	123.39	Vienna, 1930	Sch. 1,500 per pers.	Sch. 599.50	Sch. 519.48	115.40
123	18	,, 6,540	Kc. 2,777.43	Kc. 2,148.52	129.27	Winthertnur, 1922	Frs. 2,025	Kc. 3,214.41	Kc. 2,645.09	121.52

TABLE XII. — COMPARISON OF PRICES IN COPENHAGEN WITH THOSE IN 6 OTHER TOWNS IN JULY 1930

Reference No.	Number of foodstuffs considered	Total annual expenditure per unit in 1922 of Danish families whose annual consumption per man-unit $= q_0$	$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose annual consumption per man-unit $= q_1$		$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$
						Country or town and year	Expenditure			
124	22	Kr. 2,400	Kr. 186.03	Kr. 269.56	69.01	Tallinn, 1925	EKr. 400	Kr. 164.94	Kr. 276.81	59.59
125	21		,, 209.38	,, 269.51	68.61	Riga, 1926-1927	—	,, 184.90	,, 274.90	67.26
126	15		,, 223.02	,, 248.88	89.61	Belgium, 1928-1929	Frs. 5,225	,, 275.62	,, 322.18	85.55
127	18		,, 303.95	,, 255.55	117.56	Winterthur, 1922 (prices for 34 towns)	Frs. 2,025	,, 353.17	,, 325.94	108.35
128	19		,, 285.96	,, 258.56	110.60	Vienna, 1930	Sch. 1,500 per pers.	,, 293.62	,, 265.09	110.76
129	17		,, 148.79	,, 210.78	70.59	Bulgaria, 1927	Levas 14,850	,, 96.58	,, 138.51	69.73

TABLE XIII. — SUMMARY OF ELEMENTS DETERMINING THE COMPARABILITY OF DIFFERENT TOWNS WITH PRAGUE, BERLIN, WARSAW AND COPENHAGEN

towns or countries " 1 "	Values of D obtained in the comparisons between and towns " 0 "							
	Copenhagen		Prague		Berlin		Warsaw	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
Stockholm	— 8.47	— 13.08	— 9.05	— 12.67	— 9.27	— 21.21	— 21.57	— 32.56
Oslo	+ 1.04	— 3.07	— 8.77	— 11.06	— 1.92	— 5.86	— 11.25	— 16.09
Copenhagen	—	—	— 5.18	— 10.03	— 1.05	— 4.27	— 0.20	— 3.84
Prague	— 5.18	— 10.03	—	—	— 1.38	— 6.61	— 8.42	— 13.43
Berlin	— 1.05	— 4.27	— 1.38	— 6.61	—	—	+ 1.68	— 10.04
Warsaw	— 0.20	— 3.84	— 8.43	— 13.43	+ 1.68	— 5.26	—	—
	Average values							
Brussels	— 4.53		— 14.19		— 5.89		— 3.29	
Switzerland	— 7.83		— 6.00		— 6.52		— 10.38	
Vienna	+ 0.14		— 6.48		— 4.57		— 8.56	
Bulgaria	— 1.22		— 13.71		— 22.16		— 16.08	
Tallinn	— 1.97		— 18.12		— 12.37		— 0.41	
Riga	— 13.65		— 11.73		— 10.29		— 1.21	

centre—which would hardly be convenient owing to the bad comparability between Stockholm and Warsaw—at least a sort of subcentre in order to bring Tallinn and Riga into the comparison.

We may now turn to the possibilities in system II, in which Oslo forms the centre for the North and in which the comparability between centres should be better than in system I. The two towns which compare well both with Oslo and with each other are Copenhagen and Berlin. It would also be interesting to adopt for this system II a suggestion contained in the note to page 37, that is to say, to have a principal centre surrounded by secondary centres, which would merely have to fulfil the condition of comparing well with the principal centre without necessarily comparing very well with each other. The object of adopting this idea, with Oslo as principal centre, is to study the influence of such an arrangement on the final indexes. It makes it possible to neglect the relative badness of the comparison between Prague and Berlin, and to take as secondary centres Berlin, with Belgium (which is not associated with Warsaw for climatic reasons), Switzerland and Vienna on the one hand, and Prague, with Warsaw on the other. Tallinn and Riga will be compared only through the medium of Warsaw as in system I[1].

The consideration of the comparability relations has thus led to the formation of the following regions:

System I:

First region: Centre: *Stockholm.*
Other towns: Helsinki, Oslo.

Second region: Centre: *Copenhagen.*
Other towns: Berlin, Brussels and Warsaw (with Tallinn and Riga).

Third region: Centre: *Prague.*
Other towns: Vienna, " Switzerland. "

System II:

Principal centre:

Oslo: Towns associated with principal centre: Stockholm, Copenhagen, Helsinki.

Secondary centres:

Berlin, with Brussels, Vienna and Switzerland.
Prague, with Warsaw (with Tallinn and Riga).

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[1] Bulgaria has not been considered in this discussion; it appears to be comparable only with Copenhagen (see tables XII and XIII). The comparison with Vienna is much less satisfactory; in this case $P_0 = 58.78$, $P_1 = 52.65$ and $D = -10.43$. The lack of more complete material making it difficult to decide, no conclusions are drawn in this connection.

VARIOUS SPECIAL CASES

Before proceeding to discuss the formula to be adopted for the final indexes to be used for comparisons within each region and between centres, there are two questions of some importance to consider. The first is whether, in view of the fact that the preceding analyses have related only to one town in each country, it would not be necessary to consider separately the comparability of localities situated near the frontier of the country to which they belong, and possibly to assign them to another region. The second is to determine, within the limits of the available data, to what extent the date of the consumption data affects the comparability relations.

The first question is closely bound up with a point already discussed[1], namely that a positive value of D (resulting from positive correlation) should not necessarily be considered as an indication of non-comparability. It is obviously impossible to establish general rules on this subject. But it is clear that two situations, 0 and 1, cannot be considered non-comparable if r, $\frac{\sigma_p}{P_0}$ and $\frac{\sigma_q}{Q_0}$ are all very small. For, as previously shown, this combination can merely lead to the conclusion of almost complete identity between the two situations. In such a case it is evidently a matter of indifference whether r is positive or negative.

To illustrate this affirmation the results of tests made on the basis of the data for the town of Prague are given. They relate to the quantities consumed and average prices actually paid[2] by 27, 45 and 61 working-class families living in Prague whose household accounts were kept for periods of the same length (52 weeks) in successive years. The families which kept their accounts during the periods ending between July and June of the following year in 1927-28, 1928-29 and 1929-30 respectively have been grouped. The calculation of formula (i) on the basis of 24 articles first for 1928-29 in relation to 1927-28, then for 1929-30 in relation to 1928-29, and finally for 1927-28 in relation to 1929-30, gives the following results:

Reference No.		P_0	P_1	D
132	1927-1928—1928-1929	99.11	99.05	— 0.0675 per cent
133	1928-1929—1929-1930	99.25	99.17	+ 0.0817 ,,
134	1929-1930—1927-1928	101.69	101.75	+ 0.0558 ,,

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[1] See p. 44 footnote to table VI and p. 48, footnote 2.

[2] That is to say, calculated by dividing the expenditure by the quantity.

Taking the analysis somewhat further, the following values are obtained:

	r	$\frac{\sigma_p}{P_0}$	$\frac{\sigma_q}{Q_0}$
1927-1928—1928-1929. .	— 0.195947	0.038195	0.090129
1928-1929—1929-1930. .	+ 0.121680	0.080798	0.083095
1929-1930—1927-1928. .	+ 0.039173	0.111340	0.127899

As in all three cases the families belonged to the same social class, had similar incomes [1], and lived in the same town during three successive years when prices did not undergo serious modification, it is certainly legitimate to consider that these families—which, moreover, were to some extent identical during the three periods—had the same tastes and were in every respect in practically the same situation. Now it is seen that r, $\frac{\sigma_p}{P_0}$, and $\frac{\sigma_q}{Q_0}$ are relatively small in every case, and that r is negative in one case but positive in the other two. It is obvious that this positive sign does not indicate non-comparability [2].

It has been thought necessary to confirm this assertion by means of examples, because it plays a certain part in comparisons between towns belonging to the same country. The tests of this kind that have been made have in fact given the following results (see table XIV):

(1) In the comparison of Stockholm prices in July 1930 with Göteborg and Malmö prices, using for each town the quantities consumed in 1923, the value of *D* becomes positive though small, whether the calculation is based on average incomes (reference Nos. 135, 138) or the lowest income is taken for Stockholm (reference No. 137). But it is sufficient to remove three articles[3] from the list of the 23 foodstuffs used for the Stockholm-Göteborg comparison and two [4] from the 28 used for Stockholm-Malmö to obtain negative values for *D* (reference Nos. 136, 139).

(2) The substitution of Göteborg for Stockholm in the comparison with Helsinki hardly changes the value of *D* (reference

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[1] The average annual income per man-unit was as follows:

	Kc.
1927-1928	7020.04
1928-1929	7230-86
1929-1930	7970.36

[2] This is an important qualification to the second condition of comparability given on page 18.

[3] Veal, tea and cocoa.

[4] Black bread and fresh fish.

EFFECT OF THE DATE OF THE QUANTITY DATA ON COMPARABILITY

Reference No.	Number of foodstuffs considered	Total annual expenditure per unit in the year indicated of families whose consumption per man-unit = q_0		$\Sigma p_1 q_0$	$\Sigma p_0 q_0$	$P_0 \times 100$	Total annual expenditure per unit in the year indicated of families whose consumption per man-unit = q_1		$\Sigma p_1 q_1$	$\Sigma p_0 q_1$	$P_1 \times 100$	$D \times 100$
		Town and year	Expenditure				Town and year	Expenditure				
135	23	Stockholm, 1923	Kr. 1,570	Kr. 297.00	Kr. 314.06	94.57	Göteborg, 1923	—	Kr. 285.85	Kr. 301.50	94.81	+ 0.25
136	20		,, 1,570	,, 286.64	,, 300.59	95.36			,, 278.07	,, 291.79	95.30	— 0.06
137	20		,, 1,140	,, 250.49	,, 263.40	95.10			,, 278.07	,, 291.79	95.30	+ 0.21
138	28	Stockholm, 1923	,, 1,570	,, 302.85	,, 321.35	94.24	Malmö, 1923	—	,, 323.53	,, 331.11	97.71	+ 3.68
139	26			,, 262.99	,, 289.06	90.98			,, 233.33	,, 256.71	90.89	— 0.10
140	22	Denmark, 1922 (Copengagen prices)	,, 2,400	,, 299.76	,, 269.56	111.20			,, 312.80	,, 287.22	108.91	— 2.06
141	24	Germany, 1927-1928 (Berlin prices)	RM. 1,200	RM. 277.66	RM. 267.62	103.75			RM. 356.78	RM. 361.41	98.72	— 4.85
142	20	Finland, 1921	M. 6,300	Kr. 340.58	Kr. 273.18	124.67	Göteborg, 1923	—	Kr. 277.83	Kr. 241.85	114.88	— 7.85
143	23	Germany, 1927-1928 (Berlin prices)	RM. 700	RM. 233.70	RM. 217.97	107.22			RM. 315.47	RM. 364.58	86.53	—19.30
144			,, 1,200	,, 271.90	,, 266.28	102.11						—15.26
145			,, 3,400	,, 316.59	,, 324.43	97.58						—11.32
146	23	Warsaw, 1927	Zl. 835	Zl. 340.76	Zl. 361.01	94.39	Upper Silesia, 1927 (Katowice prices)	Zl. 900	Zl. 340.41	Zl. 366.17	92.97	— 1.50
147	21	Germany, 1927-1928 (Berlin prices)	RM. 1,200	RM. 172.16	RM. 264.38	65.12			RM. 142.32	RM. 217.06	65.56	+ 0.68
148	20	Germany, 1927-1928 (Berlin prices)	RM. 1,200	RM. 183.78	RM. 277.98	66.11	Warsaw, 1927	Zl. 835	RM. 168.10	RM. 259.76	64.71	— 2.12
149							Warsaw, 1928	,, 878	,, 170.83	,, 263.37	64.86	— 1.89
150							Warsaw, 1929	,, 915	,, 172.48	,, 264.65	65.17	— 1.42
151	23	Tallinn, 1925	EKr. 400	Zl. 480.41	Zl. 398.51	120.55	Warsaw, 1927	,, 835	Zl. 370.94	Zl. 308.98	120.05	— 0.41
152							Warsaw, 1929	,, 915	,, 379.56	,, 318.08	119.33	— 1.01
153	23	Riga, 1926-1927	—	,, 492.12	,, 458.72	107.28	Warsaw, 1927	,, 835	,, 373.38	,, 352.27	105.99	— 1.20
154							Warsaw, 1929	,, 915	,, 382.48	,, 364.79	104.85	— 2.27
155		Stockholm, 1923	Kr. 1,570	Kr. 205.94	Kr. 280.28	73.48	Warsaw, 1927	,, 835	Kr. 142.94	,, 268.96	53.15	—27.67
156							Warsaw, 1929	,, 915	,, 141.27	,, 265.81	53.15	—27.67

No. 142 and table VI); nor does the comparability of Göteborg differ from that of Stockholm in the comparison with Berlin, *D* varying between — 11.32 and — 19.30 per cent., as compared with — 9.27 and — 18.93 per cent. (reference Nos. 143-145 and table VI)[1].

(3) On the other hand, the substitution of Malmö for Stockholm in the comparisons with Copenhagen and Berlin (reference Nos 140, 141 and table VI) substantially modifies the values of *D*: — 2.06 is obtained instead of — 8.47 and — 13.08 per cent., and — 4.85 instead of — 9.27 and — 18.93 per cent.

(4) In the comparison of Katowice prices in July 1930 with Warsaw prices, using the quantities consumed in 1927 in Upper Silesia and at Warsaw, *D* becomes very small and negative (reference No. 146).

(5) By substituting Katowice for Warsaw in the comparison with Berlin *D*, which was — 2.12 per cent. for the two average incomes at Warsaw and Berlin but otherwise varied between + 1.68 and — 5.26 per cent., becomes + 0.68 per cent. (reference No. 147). As was done above[2] when *D* was known for only one combination of incomes and was positive, this case is considered one of non-comparability.

It may therefore be concluded that no general statement can be made on the effect on comparability of substituting one town for another belonging to the same country in the comparison with a town of another country. The same applies to comparability between towns in the same country. In particular, it seems that Göteborg might well be left in the same region as Stockholm, while Malmö would be better associated with Copenhagen. As regards Katowice, the rare results available suggest that direct comparison with Warsaw is preferable to that with Berlin.

The second question concerning the effect of the date of the quantity data has already been gone into in an earlier enquiry of the Office[3]. The object then was to ensure that the result of formula (i) would be the same irrespective of the date of the quantity data. This condition was satisfied by the example given, for the index of prices obtained by formula (i) was practically the same in the comparison of Antwerp prices with Detroit prices, whether the weights were the quantities consumed in 1908-1909, or those in 1921, or those in 1928-1929. The values of D, however, are not the same in the three cases. The following results are obtained:

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[1] The smaller variability in the Berlin-Göteborg comparison may be partly ascribed to the fact that only the general average of the quantities consumed at Göteborg could be used for these calculations.

[2] See p. 48 footnote 2.

[3] " Contribution ", p. 224, table II.

Date of quantity data	P_0	P_1	D
1908-1909	73.22	67.07	— 8.40
1921	79.04	64.42	— 18.50
1928-1929	74.31	63.12	— 15.06

It follows from these few figures that it can hardly be assumed that the comparability relations remain stable for long periods. On the other hand, to judge from the examples given in table XIV (reference Nos. 148-156), it seems that the use of the data collected at Warsaw during three successive years has not much influence on comparability. As regards the result of formula (i), it remains practically unaffected, as was already the case in the comparison Detroit-Antwerp previously mentioned.

From the practical point of view, the examples given show that it would be advisable to reconsider any system of regions whenever new data are available, and in any case every three or five years.

IV

ESTABLISHMENT OF FINAL INDEXES FOR JULY 1930

CHOICE OF FORMULAE FOR CURRENT COMPARISONS

In the preceding discussion only formula (i) has been used. Only by means of this special formula has it been possible to formulate the ideal conditions for good comparability and to adopt a practicable method, from the point of view of the work involved, for actually constituting the region. Nevertheless, the exclusive use of this formula for current comparisons suffers from two drawbacks:

(1) It does not satisfy the circular test;

(2) its application, even with the simplifications rendered possible by the comparison by regions, involves a great deal of calculation.

To meet these two objections a formula of the nature of formula (iv) will be adopted for comparisons within regions or, rather, between towns in the same region and their centre. In other words, for this type of comparison recourse is had to the former practice of the Office of considering as an index of prices between two towns the ratio of the cost of a " basket of provisions ", defined for each town on the basis of current prices. The fixed weights chosen for each region will be the simple averages of the general averages of the quantities of each article consumed annually per unit of consumption in each of the towns composing the region [1]. These indexes therefore satisfy the circular test, and comparisons can be made between towns in the same region by dividing the known indexes for each town in relation to its centre (taken as base = 100) by each other. As the towns have been grouped in regions in such a way as to make D as small and stable as possible, so that P_0 and P_1 are relatively little affected by the standard of living in all the direct comparisons involved in the system, the indexes of formula (iv) will not differ much from those obtained from formula (i)[1], as will be demonstrated in practice below.

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[1] No account is therefore taken of the results of the two fuller analyses presented in tables I and II. It would in fact be impossible to use them, since in the circumstances comparable data for other comparisons are not available.

For the comparisons between centres, it will be remembered that the first is based primarily on good comparability within the regions, while the second is obtained by attaching most importance to comparability between the secondary centres and the principal

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[1] When a comparison is made between two towns only, it can be shown that

$$> P_2 \gtrless P_1 ,$$

where $P_0 = \dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$; $P_2 = \dfrac{\Sigma (q_0 + q_1) p_1}{\Sigma (q_0 + q_1) p_0}$; $P_1 = \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1}$.

For

$$P_0 - P_2 = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} - \frac{\Sigma (q_0 + q_1) p_1}{\Sigma (q_0 + q_1) p_0}$$

$$= \frac{\Sigma p_1 q_0 \Sigma p_0 q_0 + \Sigma p_1 q_0 \Sigma p_0 q_1 - \Sigma p_0 q_0 \Sigma p_1 q_0 - \Sigma p_0 q_0 \Sigma p_1 q_1}{\Sigma p_0 q_0 \Sigma (q_0 + q_1) p_0} .$$

If $\Sigma (q_0 + q_1) p_0$ is neglected in the denominator as being always positive,

$$\begin{aligned} P_0 - P_2 &= \Sigma p_1 q_0 + \Sigma p_0 q_1 \cdot P_0 - \Sigma p_0 q_0 \cdot P_0 - \Sigma p_1 q_1 \\ &= - \Sigma p_1 q_1 + P_0 \cdot \Sigma p_0 q_1 , \text{ since } P_0 \cdot \Sigma p_0 q_0 = \Sigma p_1 q . \end{aligned}$$

(x) Thus $P_0 - P_2 = \Sigma p_0 q_1 (P_0 - P_1)$.

Similarly

$$P_1 - P_2 = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} - \frac{\Sigma (q_0 + q_1) p_0}{\Sigma (q_0 + q_1) p_0}$$

$$= \frac{\Sigma p_1 q_1 \Sigma p_0 q_0 + \Sigma p_1 q_1 \Sigma p_0 q_1 - \Sigma p_0 q_1 \Sigma p_1 q_0 - \Sigma p_0 q_1 \Sigma p_1 q_1}{\Sigma p_0 q_1 \Sigma (q_0 + q_1) p_0}$$

which, neglecting $\Sigma (q_0 + q_1) p_0$ in the denominator as always positive,

$$\begin{aligned} &= \Sigma p_0 q_0 P_1 + \Sigma p_1 q_1 - \Sigma p_1 q_0 - \Sigma p_0 q_1 P_1 \\ &= - \Sigma p_1 q_0 + P_1 \cdot \Sigma p_0 q_0 , \quad \text{since } P_1 \cdot \Sigma p_0 q_1 = \Sigma p_1 q_1 . \end{aligned}$$

(xi) Thus $P_1 - P_2 = \Sigma p_0 q_0 (P_1 - P_0)$.

It follows from expressions (x) and (xi) that $P_0 - P_2$ and $P_1 - P_2$ are necessarily opposite in sign, both in the general case when $P_0 > P_1$, and in the particular cases when $P_1 > P_0$. Therefore $P_0 - P_2 \gtrless 0$ and $P_1 - P_2 \lesseqgtr 0$, so that

$$P_0 \gtrless P_2 \gtrless P_1 \qquad \textit{Q.E.D.}$$

We are indebted to Professor Bowley for this proof.

centre. It would therefore be advisable to make allowance for the smaller comparability between centres, at least in system I, and the simplest method of doing so is, if not to take the values of P_0 and P_1 concurrently for the inter-regional comparisons, at all events to provide each index with maximum and minimum limits between which the values of P_0 and P_1 should lie[1].

In the two systems, therefore, we have indexes expressing the ratio of prices between each town and its centre, the latter being taken as base, of the form $\frac{\Sigma p_1 q}{\Sigma p_0 q}$ where the q's are the averages of the quantities consumed for all the towns belonging to the region. It is obvious that the list of foodstuffs considered, while being as long as possible, must be limited to articles of common consumption. Further, it will be the same for all the towns of the region and will constitute a regional " basket of provisions ".

For the comparisons between centres, use will be made in both systems of formula (i), subject to the limits within which P_0 and P_1 must lie. The formula will thus be

$$\sqrt{P_0 \cdot P_1} \pm (P_0 - \sqrt{P_0 \cdot P_1}),$$

which is nearly equal to

$$\sqrt{P_0 \cdot P_1} \pm (P_1 - \sqrt{P_0 \cdot P_1});$$

that is to say, the limits are expressed in terms of the same base as the index itself[2].

RESULTS OBTAINED BY APPLYING THE CHOSEN FORMULAE

Tables XV to XVIII give the regional and inter-regional indexes obtained by applying the formulae adopted above for the two systems of regions, I and II.

Concerning first the regional indexes of the two systems

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[1] It might be asked why, in indicating limits, a reservation is made only for the indexes relating to comparisons between centres. The observation is perfectly legitimate. The reason why it has not been taken into account is simply that the regions are composed of towns between which comparisons are good enough to allow of making use of one index only. Hence, this precaution might be dispensed with also for comparisons between the secondary centres and the principal centre of system II.

[2] The reason why the geometric and not the simple average of P_0 and P_1 has been taken is that indexes of the latter kind do not satisfy the reversibility test.

(tables XV and XVII), it should be noticed that most of the budgets, on the basis of which the indices have been compiled, contain 22 foodstuffs. This number is higher in a few cases than that taken for formula (i), for it was held possible to make certain reasoned estimates either of particular prices or of particular quantities. There can be no doubt that the statistical authorities of the countries concerned would be able to supply the missing data if specially asked to do so. In any case, none of the estimates are very uncertain, and they cannot be expected to have an important effect on the result[1].

Further, it appears from the comparison of the values of P_0 and P_1 obtained in the calculation of formula (i) between each town and its centre (columns 4 and 5) with the result obtained by the regional budget (column 8) that the latter generally lies well between P_0 and P_1. This is so in spite of the fact that for calculating the regional budgets or baskets use has been made of more recent quantity data for Prague and Warsaw than those used in the calculation of P_0 and P_1. These data were, in fact, not published until after the completion of the calculations. The only important exception in system I is Belgium, where the regional index considerably exceeds the value of P_0. Now it will be remembered that Belgium was assigned to the Copenhagen region on the basis of a single value of D. It will therefore be interesting to see whether, by calculating other values for D of the basis of data relating to other income classes both at Copenhagen and at Brussels, the conclusion will be that comparability between these two towns can be considered good or bad on the whole. The results of this calculation, based on 17 articles, are as follows:

Reference No.	Income per unit at Copenhagen in 1922	P_0	Income per unit in Belgium in 1928-1929	P_1
	Kr.		Fr.	
157	1,600 and under	87.91	13,800	81.92
158	2,400	89.61	16,700	85.50
159	3,200 and over	93.25	22,000	89.17

The values of D that can be obtained from these figures vary between + 1.43 and — 12.15 per cent. The case would therefore have been considered very unfavourable for direct comparison were it not that it was held sufficient (see table XIII) for Brussels and certain other towns to have only one value of D in order to reduce the work of calculation.

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[1] For details see the tables given in Appendix IV, pp. 104-105.

In spite of the unsatisfactory comparability between Copenhagen and Brussels indicated by the above figures, the fact of excluding the Belgian quantities from the Copenhagen regional budget has little influence on the other indexes for this region, as will appear from the following table:

INDEXES FOR COPENHAGEN WHEN BELGIAN QUANTITIES ARE

	Included	Excluded
Berlin.	112.09	113.45
Warsaw	74.49	76.22
Riga	70.15	71.78
Tallinn	62.59	64.04

In view of this small effect, it has been held a matter of indifference whether Brussels is included or excluded in system I. It has in fact been included. Moreover, it is very probable that if the list of countries considered were more complete — and it is hoped that this will be the case in the near future thanks to new budget enquiries -Brussels would be assigned to another region. The question is thus of comparatively little importance. It explains, however, the very marked differences between the two systems for all indexes involving Brussels.

Among the regional indexes of system II, mention may be made of the only exception, the Copenhagen index (Oslo region), though it is much less important than that of Brussels in system I. Here too the regional index is higher than P_0. This necessarily affects all the indexes involving Copenhagen, without, however, being of outstanding importance as compared with other divergences between the final results obtained by the two systems.

Table XV has been completed by results obtained in the past by the " international basket " method for July 1930 [1]. It shows that the new regional indexes sometimes give results differing substantially from the old international indexes. In table XVII these indexes have been omitted, comparisons on the old methods being available in only one case.

For the inter-regional indexes formula (i) has been used subject to limits within which P_0 and P_1 must lie. It should be observed that

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[1] See *International Labour Review*, Vol. XXII, No. 4, October 1930, and Vol. XXVII, No. 4, April 1933.

TABLE XV. — RESULTS OBTAINED BY APPLYING DIFFERENT FORMULAE TO THE COMPARISON OF PRICES BY REGIONS IN SYSTEM I FOR JULY 1930

Centre	Other towns	Indexes with the centre taken as base = 100, obtained by								
		formula (i)				regional basket		international basket		
		Number of foodstuffs considered	P_0	P_1	Index	Number of foodstuffs considered	Index	Number of foodstuffs considered	Number of towns considered	Index
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Stockholm . . .	Oslo	26	108.22	101.78	104.95	22	106.20	—	—	—
	Helsinki.	23	82.14	76.27	79.15		79.29	—	—	—
Prague	Vienna	19	123.39	115.40	119.33	18	121.33	14	3	106.62
	Switzerland . . .	18	129.27	121.52	125.34		127.74	—	—	—
Copenhagen . . .	Berlin.	24	112.82	109.96	111.38	22	112.09	14 (25)	6 (6)	118.88 (105.34)
	Brussels.	17	91.36	87.13	89.22		97.03	—	—	—
	Warsaw.	18	76.40	75.38	75.88		74.49	14 (25)	4 (4)	72.03 (67.48)
	Riga (via Warsaw)	23	72.37	70.73	71.55		70.15	—	—	—
	Tallinn (via Warsaw) .	23	63.59	62.94	63.26		62.59	14	2	65.03

TABLE XVI. — RESULTS OBTAINED BY APPLYING DIFFERENT FORMULAE TO THE COMPARISON OF PRICES AT THE CENTRES OF SYSTEM I IN JULY 1930

Centre, prices taken as base " 0 "	Index of prices, obtained by applying formula (I) for the three centres					
	Stockholm		Copenhagen		Prague	
	Number of foodstuffs considered	Index	Number of foodstuffs considered	Index	Number of foodstuffs considered	Index
Stockholm	—	—	20	95.26 ± 5.57	17	91.16 ± 5.39
Copenhagen	20	105.01 ± 6.18	—	—	17	95.09 ± 3.55
Prague	17	109.69 ± 6.48	17	105.17 ± 3.92	—	—

TABLE XVII. — RESULTS OBTAINED BY APPLYING DIFFERENT FORMULAE TO THE COMPARISON OF PRICES BY REGIONS IN SYSTEM II, FOR JULY 1930

Centre	Other towns	Indexes, with the centre taken as base = 100, obtained by					
		formula (i)				regional basket	
		Number of foodstuffs considered	P_0	P_1	Index	Number of foodstuffs considered	Index
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Oslo	Stockholm	22	98.30	92.42	95.31	22	94.49
	Copenhagen	19	87.53	86.23	86.88		90.84
	Helsinki	18	78.78	67.99	73.19		75.66
Berlin	Vienna	21	101.76	97.11	99.40	20	102.26
	Switzerland	20	109.13	102.02	105.52		106.98
	Brussels	15	82.70	77.86	80.24		82.59
Prague	Warsaw	17	81.39	72.66	76.90	19	77.64
	Riga (via Warsaw)	23	73.34	71.68	72.51	22	73.11
	Tallinn (via Warsaw)	23	64.44	63.79	64.11		65.23

TABLE XVIII. — RESULTS OBTAINED BY APPLYING DIFFERENT FORMULAE TO THE COMPARISON OF PRICES AT THE CENTRES OF SYSTEM II IN JULY 1930

Centre taken as base " 0 "	Index of prices, obtained by applying formula (i)					
	Oslo		Berlin		Prague	
	Number of foodstuffs considered	Index	Number of foodstuffs considered	Index	Number of foodstuffs considered	Index
Oslo	—	—	22	95.51 ± 2.29	16	84.41 ± 4.71
Berlin	22	104.70 ± 2.52	—	—	16/22 18	88.38 ± 7.17 86.43 ± 2.07
Prague	16	118.47 ± 6.61	22/16 18	113.15 ± 9.17 115.71 ± 2.77	— —	— —

although the formula itself allows of reversing the base, this is not so for the indication of the limits. For these have been calculated on the basis of the formula $\pm\ (P_0 - \sqrt{P_0 P_1})$, rather than $\pm \frac{(P_0 - \sqrt{P_0 P_1})}{\sqrt{P_0 \,.\, P_1}}$, because the former facilitates linking up the regional indexes with the inter-regional indexes. It is, in fact, sufficient to multiply both the result of formula (i) and the limits by the regional index to obtain the desired transposition of the latter. Further, it is well known that formula (i) does not satisfy the circular test.

For system I, indexes are given connecting the three centres. Although, as will be remembered, the values of D for these indexes are on the average fairly high, they are nevertheless sufficiently independent of the standard of living to allow of a direct comparison between all these three centres.

It is not so in system II. Here the direct comparison between Prague and Berlin is fairly bad. On the other hand, both these towns compare well with Oslo taken as principal centre. For this reason it is proposed to avoid direct comparisons between the Prague and Berlin regions by passing each time through Oslo. It is thought necessary, however, to show the effect of two possibilities; and two indexes are accordingly given for Prague-Berlin and for Berlin-Prague in table XVIII, one obtained by way of Oslo [1], and the other giving formula (i), calculated directly for these two combinations.

Finally, table XIX gives the indexes of prices that can be calculated between all the towns considered according to the two systems. Further, for system II two indexes are given for all the combinations involving the comparison Prague-Berlin or Berlin-Prague. To simplify the work, only the indexes $\frac{\text{town } 1}{\text{town } 0}$, but not

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[1] If p_0, q_0 represent Oslo prices and quantities,

p_P, q_P those for Berlin,
p_P, q_P those for Prague

these indexes may be written as follows:

$$\text{Index } \frac{\text{Prague}}{\text{Berlin}} = \sqrt{\left(\frac{\Sigma q_0 p_P}{\Sigma q_0 p_0} \cdot \frac{\Sigma q_B p_0}{\Sigma q_B p_B}\right) \cdot \left(\frac{\Sigma q_P p_P}{\Sigma q_P p_0} \cdot \frac{\Sigma q_0 p_0}{\Sigma q_0 p_B}\right)}\,.$$

For the calculation of the limits, the first bracket under the square root has been taken as P_0.

TABLE XIX. — INDEXES OF PRICES CALCULATED BY MEANS OF TWO SYSTEMS OF REGIONS

Locality taken as base = 100		Locality taken as town "1"											
		Stockholm	Oslo	Helsinki	Copenhagen	Berlin	Brussels	Warsaw	Riga	Tallinn	Prague	Vienna	Switzerland
Stockholm	I	100	106.20	79.29	95.26 ± 5.57	106.78 ± 6.24	92.43 ± 5.40	70.96 ± 4.15	66.82 ± 3.91	59.62 ± 3.49	91.16 ± 5.39	110.60 ± 6.54	116.45 ± 6.89
	II		105.83	80.07	96.14	101.08 ± 2.42	83.48 ± 2.00	69.36 ± 3.87	65.31 ± 3.64	58.27 ± 3.25	89.33 ± 4.98	103.36 ± 2.47	108.14 ± 2.59
Oslo	I		100	74.66	89.70 ± 5.24	100.55 ± 5.88	87.03 ± 5.08	66.82 ± 3.91	62.92 ± 3.68	56.14 ± 3.29	85.84 ± 5.08	104.14 ± 6.16	109.65 ± 6.49
	II			75.66	90.84	95.51 ± 2.29	78.88 ± 1.89	65.54 ± 3.66	61.71 ± 3.44	55.06 ± 3.07	84.41 ± 4.71	97.67 ± 2.34	102.18 ± 2.45
Helsinki	I			100	120.14 ± 7.02	134.67 ± 7.87	116.67 ± 6.81	89.49 ± 5.23	84.27 ± 4.93	75.19 ± 4.40	114.97 ± 6.80	139.49 ± 8.25	146.87 ± 8.69
	II				120.06	126.24 ± 3.03	104.26 ± 2.50	86.62 ± 4.84	81.56 ± 4.55	72.77 ± 4.06	111.56 ± 6.23	129.09 ± 3.09	135.05 ± 3.24
Copenhagen	I				100	112.09	97.03$_1$	74.49	70.15	62.59	95.09 ± 3.55	115.37 ± 4.31	121.47 ± 4.53
	II					105.14 ± 2.52	86.14 ± 2.52	72.15 ± 4.03	67.93 ± 3.79	60.61 ± 3.38	92.92 ± 5.18	107.52 ± 2.58	112.48 ± 2.70
Berlin	I					100	86.56	66.46	62.58	55.84	84.83 ± 3.17	102.93 ± 3.85	108.37 ± 4.04
	II *a*						82.59	68.62 ± 5.57	64.61 ± 5.24	57.65 ± 4.68	88.33 ± 7.17	—	—
	II *b*							67.10 ± 1.61	63.19 ± 1.51	56.38 ± 1.35	86.43 ± 2.07	102.26	106.98
Brussels	I						100	76.77	72.30	64.51	98.00 ± 3.66	118.90 ± 4.44	125.19 ± 4.67
	II *a*							83.09 ± 6.74	78.23 ± 6.34	69.80 ± 5.67	107.01 ± 8.68	—	—
	II *b*							81.24 ± 1.95	76.51 ± 1.83	68.26 ± 1.63	104.65 ± 2.51	123.82	129.53
Warsaw	I							100	94.17	84.02	127.65 ± 4.77	154.88 ± 5.79	163.07 ± 6.08
	II *a*								—	—	—	149.03 ± 12.08	155.91 ± 12.64
	II *b*								94.17	84.02	128.80	152.41 ± 3.65	159.44 ± 3.81
Riga	I								100	89.22	135.55 ± 5.06	164.46 ± 6.14	173.16 ± 6.46
	II *a*									—	—	158.27 ± 12.83	165.57 ± 13.42
	II *b*									89.22	136.78	161.85 ± 3.87	169.32 ± 4.05
Tallinn	I									100	151.93 ± 5.67	184.33 ± 6.80	194.07 ± 7.24
	II *a*										—	177.39 ± 14.38	185.57 ± 15.04
	II *b*										153.30	181.40 ± 4.34	189.77 ± 4.54
Prague	I										100	121.33	127.74
	II *a*											115.71 ± 9.38	121.05 ± 9.81
	II *b*											118.33 ± 2.83	123.79 ± 2.96
Vienna	I											100	105.28
	II												104.62
Switzerland													100

N.B. — Indexes I relate to system I, indexes II to system II. In addition indexes II*a* have been calculated by way of Oslo and indexes II*b* by means of direct comparison between Prague and Berlin.

the indexes $\frac{\text{town } 0}{\text{town } 1}$ have been given. It is obvious that as all the indexes (except the limits) allow of reversing the base, the lower part of the table ought to be filled in with the reciprocals of the indexes shown in the upper part.

Except for the effect, already mentioned, of the fact that the regional index for Brussels in system I and that for Copenhagen in system II have values exceeding their respective P_0's, the divergences between the indexes of the two systems are due to the following causes: (*a*) formula (i) does not satisfy the circular test; (*b*) the formula used for the regional indexes satisfies it only when the list of foodstuffs and the system of weights are identical everywhere, which is not the case for various regional baskets. It is not surprising therefore that the divergences are fairly substantial in certain cases. Nevertheless, the indexes given by one of the two systems nearly always fall between the limits of the index given by the other[1]. This demonstrates the usefulness of the innovation. It must be admitted that examination of these results scarcely allows preference to be given to the one or the other regional system. If, in the present state of research, it was desired to choose between them, the choice would be largely arbitrary. Accordingly, we think it better to refrain at present.

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[1] In view of the relatively bad comparability between Berlin and Prague, and the fact that D chances to be relatively small for the two medium incomes in these two towns, the limits shown for indexes II*b* in table XIX are too small.

V

CONCLUSIONS

In the last chapter were given the various indices to which our analyses have led. Without reverting to the theoretical considerations set out in the first part of this study and summarised on pages 29, 30, it is clear that these indices are no more than a first attempt at a practical solution of the many problems involved in the computation of index-numbers of food prices. Even from this limited standpoint, these indices are still far from being fully satisfactory. In particular:

(1) the regions from which they were obtained have not been selected in the light of research as thorough as the theoretical discussion has shown to be desirable;

(2) the number of countries considered is very small, and it is clearly regrettable that the absence of suitable budget enquiries has prevented the inclusion of these indices for countries like France, Great Britain, Italy and the Netherlands; for the same reason it has not been possible to consider overseas countries (for example, the United States of America) for which, though the necessary information is available, comparisons with Europe could not reasonably be attempted on the basis only of the countries studied here.

(3) the need of overcoming the multiple practical difficulties due to the imperfections and deficiencies of the statistical material available, and also of keeping the calculations within certain limits, compelled us to adopt procedures that call for many reservations on which we have already dealt at some length and which cannot be over-emphasised.

In spite of such defects, indices obtained by these methods are superior to those formerly obtained from a single " international basket ". The fundamental difference of character between the earlier calculations and our present researches renders unnecessary a detailed comparison of the results given by the two methods.

The present study does not in any way claim to have exhausted either the theoretical discussion or the practical consideration of the many questions successively dealt with and still awaiting elucidation. It is to be hoped that future researches and improvements in the statistical material available will enable us to throw fresh light on all these questions.

The problems inherent in a correct comparison of food prices are only a part of the larger problems inherent in the comparison of the cost of living as a whole. Investigations with a view to compiling international index numbers of rents have already been undertaken by the Office and form the subject of Part II of this study. It is hoped subsequently to deal with the comparison of heating and lighting expenses, and the combination of indices for these three heads of expenditure. If these efforts are crowned with success, an important part of the characteristic expenditure of a family budget will have been covered and an important step will have been taken toward the solution of the problem of international comparisons of the cost of living.

APPENDIX I

VARIOUS THEORIES ON THE RELATION BETWEEN PRICE INDEX NUMBERS AND THE STANDARD OF LIVING

It is proposed in this Appendix to examine the theories put forward by various writers on the subject of the relations between certain index numbers of prices, and in particular the formulae of Laspeyres (P_0) and Paasche (P_1), and the theoretical " price of living ", which is defined as the ratio between incomes securing the same standard of living, or, in other words, procuring the same sum total of economic satisfaction, in two situations which differ only in respect of prices.

These theories underlie the discussion of the second chapter of the first part of this volume. Their closer analysis in this Appendix is intended not only to lay them before the reader, but also to draw attention to the rather suggestive fact that several writers have arrived independently at essentially identical results by different methods. The existence of such a phenomenon must mean that this idea of the "limiting " quality of the Laspeyres and Paasche formulae must also have been considered more or less subsidiarily and accidentally by many writers. It is not proposed to give all their names, or to refer to the frequent allusions to this problem to be found in the whole of the literature dealing with index numbers.

All that will be done here is to reproduce as briefly as possible the more developed theories due to Pigou (1912), Konüs (1924), Haberler (1927), Bowley (1928), Keynes (1930), Bortkiewicz (1932) and Allen (1933). They will not be taken in strictly chronological order, however, but will be distinguished according to the line of argument and the results obtained. For this reason Pigou will be taken first, not because the essential parts of his theory date from 1912, but, above all, because of the special nature of his ideas. Next we shall take Haberler, comparing him with Konüs (according to Bortkiewicz). This will be followed by the theories of Bortkiewicz, the successor of Konüs, and Keynes. A few brief indications will relate to Bowley, whose theory holds a special position. Finally, we shall deal with Allen, who has given a particularly conclusive form to his exposition.

In setting forth these theories, we have sought to give them, so far as possible, a more comparable form, not only by substituting our notation for theirs, but also by abridging where this seemed justified, or amplifying where this seemed necessary, either the line of argument or the conclusions. It is hoped that this will bring out more clearly what is common to these theories, at the same time as it draws attention to their differences—without, however, modifying their essential features.

In 1912, Pigou devoted a chapter of his *Wealth and Welfare* [1] to a search for a measure of the national dividend. This measure was to be based on price changes, and to be of such a nature that its reciprocal —nominal incomes being supposed constant—would always vary in the same sense (but obviously not in the same proportion) as the total economic welfare or satisfaction of a group of persons. Now, this measure of prices may be constructed in two ways: by using the quantities consumed in the first, or those in the second situation—that is to say, the two formulae P_0 and P_1. If it is further assumed that the tastes, temperament, etc., of the community in question remain constant during the period of observation in the sense that for the representative man of the group they are identical, and that the two measures (the formulae P_0 — Laspeyres — and P_1 — Paasche —) indicate a rise in prices, it is reasonable to suppose that the welfare of this representative man, and therefore of the community, has diminished in the second situation as compared with the first. In the same way this welfare will be greater if the two formulae indicate a fall in prices. There remains the case where one of the formulae indicates an increase and the other a fall in prices, and it is at this point that the reasoning of interest here applies:

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[1] See No. 1 of the List of Sources given at the end of this Appendix, p. 91.

If the first formula (P_0 in our notation) exceeds unity, this means that if the representative man wished with the same nominal income to consume the various commodities in the second situation in the same proportion as in the first, he would have to reduce the quantity of each commodity consumed by a certain proportion. He would therefore experience a loss of satisfaction, say, by K_1. Therefore, the fact that, instead of making this proportionate reduction of the quantities (q_0', q_0'', q_0''' ...), he consumes the quantities (q_1', q_1'', q_1''' ...), proves that the satisfaction he obtains does not fall short of the satisfaction obtained from the first collection (q_0', q_0'', q_0''' ...) by more than K_1. If, further, P_1 is less than unity and the representative man wished his consumption in the first situation to be in the same proportion as in the second, he would have to reduce each quantity q'_1, q_1'', q_1''' ... by a certain proportion. He would therefore experience a loss of welfare, say, by K_2. The fact that, instead of making this proportionate reduction of the q_1's, he consumes q_0's, proves that the satisfaction be obtains does not fall short of the satisfaction obtained from the q_1's by more than K_2. In other words, restoring the chronological order, it is found that the satisfaction obtained from the q_1's does not exceed that obtained from the q_0's by more than K_2.

Although Pigou does not mention it at this point—it was not until the later editions of *Economies of Welfare* that he spoke of "limits" *after* having given the above reasoning—this line of argument obviously leads to the conclusion that the two formulae P_0 and P_1, applied separately to the determination of purchasing power of a constant nominal income, are independent variables of two functions (which may be called $K_1 = f(P_0)$ and $K_2 = \varphi(P_1)$), whose values are in turn the limits of the variation of satisfaction. It is obvious that it would be possible—and in fact this is what Pigou has done in subsequent work—to generalise the application of this theorem by the simple introduction of the ratio of the nominal incomes of the representative man in the two situations.

It was thought necessary to reproduce this argument because it contains, twelve years before the publication of the first more complete theory (that of Konüs, 1924), the essential elements of the latter. It concludes, in fact, according to the *choices* made by the "representative man" (incidentally, a concept open to question), assuming identity of tastes, that the aggregate of commodities to which preference is given procures greater satisfaction.

In his book *Der Sinn der Indexzahlen* (1927)[1], Haberler undertook to examine the possibilities of constructing an index of prices which should not be purely " mechanical ", but should be capable of economic interpretation. To this end he distinguishes between (*a*) nominal income or " money-income ", (*b*) income in the form of the goods which the individual exchanges for this money-income, or " material-income ", and (*c*) the psychological value of this material-income, that is to say the total satisfaction it procures, or " satisfaction-income ". The problem is to know whether, the money-incomes and material-incomes being known in two different situations, as also the prices in force in the two situations, it is possible to arrive at any conclusion concerning the relation between the satisfaction-incomes.

The analysis is on the following lines: Haberler first develops what he calls the two fundamental laws for the comparison of incomes:

> I. — As long as prices remain the same, any individual having to choose between two money-incomes prefers the higher.

This " law " is obviously valid only other things being equal. It is therefore assumed that all other circumstances which may influence the choice of the individual remain unchanged, in particular, tastes.

> II. — Any individual having to choose between two money-incomes in two situations which differ only in respect of prices would prefer that which is sufficient to obtain at least the material-income corresponding to the other money-income.

This second " law " is checked by a comparison of the nominal income in the second situation, $R_1 = \Sigma p_1 q_1$, with the material-income of the first situation (q_0', q_0'', q_0''' ...) valued at the prices of the second, namely, $\Sigma p_1 q_0$. If it is found, for instance, that $\Sigma p_1 q_1 > \Sigma p_1 q_0$, or, which is the same thing, $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} > \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$, that is, $\frac{R_1}{R_0} > P_0$, then the income R_1 will be preferred to the income R_0. This will be so even if $\Sigma p_1 q_1 = \Sigma p_1 q_0$, that is, $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$, or $\frac{R_1}{R_0} = P_0$; for the fact that the individual, instead of consuming (q_0', q_0'', q_0''' ...), consumes (q_1', q_1'', q_1''' ...), proves that he derives greater satisfaction from the collection of q_1's than from the collection of q_0's, although the money income R_1 would have been quite sufficient to allow him to consume the latter.

If the change in the level of prices for the individual in question is defined as the change in money-income needed to procure the same satisfaction-income, it follows that the change in P_0 will always be greater than this change in the level of prices. The choice made by the individual between R_1 and R_0 is indifferent only on condition that

$$\frac{R_1}{R_0} < P_0. \tag{1}$$

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[1] List of Sources, No. 3.

If $\frac{R_1}{R_0} \geqq P_0$, he would prefer R_1.

Now, it is clear that condition (1) is not sufficient to guarantee indifference of choice or, in other words, equivalence of money-incomes, that is, equality of satisfaction-incomes; for it would be possible for R_1 to procure less satisfaction than R_0.

Hence Haberler proceeds to a further demonstration. He assumes that, for instance, $\Sigma p_0 q_1 < \Sigma p_1 q_1$, which indicates that the material-income $(q_1', q_1'', q_1''' ...)$ would cost less at the prices of situation 0 than at the prices of situation 1. If, therefore, the money-income R_0 of the individual in situation 0 bears a higher ratio to the money-income R_1 than $\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}$, he would certainly prefer R_0 to R_1. His choice would still be the same if $\frac{R_0}{R_1} = \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}$, for he proves by the fact that he consumes the q_0's — and not the q_1's as his income would allow him to do—that he derives greater satisfaction from the consumption of the collection of q_0's. In order that his choice may be indifferent, it is therefore necessary that $\frac{R_0}{R_1} < \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}$, in other words

$$\frac{R_1}{R_0} > P_1. \tag{2}$$

This leads to the conclusion that P_1 provides another condition for the equivalence of money-incomes or the equality of satisfaction-incomes. And this is precisely the condition which was wanting to guarantee that R_1 should not fall short of R_0 in satisfaction.

It thus follows that for one and the same individual whose tastes remain unchanged during the period of observation, and for whom all other circumstances capable of affecting his choice, except prices, also remain unchanged, P_0 and P_1 indicate the limits of variation in the level of prices. In other words, P_0 and P_1 measure an interval within which the coefficient must lie by which income R_0 must be multiplied to find an income R_1 equivalent to it.

Haberler's argument calls, however, for certain precisions. It is evident that the composition of the material-incomes varies very considerably with the money-income. It is hardly legitimate to assume —especially in a completely non-numerical treatment such as Haberler's —that P_1 will not be affected by the amount of the money-income in situation 1 to which the data $(q_1', q_1'', q_1''' ...)$ apply. To obtain a satisfactory solution of his problem, it would therefore be necessary to know the money-income R_1 which is equivalent, or at least nearly equivalent, to R_0 (as Haberler requires) before proceeding to its search by the use of P_0 and P_1. Haberler, without raising the question in these terms, confines himself to requiring that the two satisfaction-incomes (by mistake he speaks here of money-incomes, p. 96) considered in the two countries should not be too different.

Three years before the publication of Haberler's book his theory had already been developed by a Russian writer, A. A. Konüs (unknown to Haberler), as Bortkiewicz points out in a critical account of his *Sinn der*

Indexzahlen[1]. It is specially noteworthy that Konüs's argument, which Bortkiewicz describes as " final and irrefutable ", already contained a serious criticism of Haberler's theory. It is in brief as follows: It is perfectly correct to say that P_0 always gives the upper limit of the change in the cost of living, but it is not always true that P_1 is the lower limit.

The reasoning leading to the conclusion that P_0 is always the upper limit of the change in the cost of living rests on the consideration that a money-income of the amount $P_0 \,.\, R_0 = \Sigma p_1 q_0$ represents in situation 1 a satisfaction-income higher than that corresponding to the money-income $\Sigma p_0 q_0 = R_0$ in situation 0, because it would enable the individual, not only to consume $(q_0', q_0'', q_0''' \ldots)$, but by redistributing his expenditure in a modified price system, to attain greater satisfaction.

A similar line of reasoning leads to the corresponding conclusion for P_1. It is based on the consideration that a money-income of the amount $\frac{R_1}{P_1} = \Sigma p_0 q_1$ always represents in situation 0 a satisfaction-income higher than that corresponding to the money-income $\Sigma p_1 q_1$ in situation 1 because it would enable the individual not only to consume $(q_1', q_1'', q_1''' \ldots)$, but by redistributing his expenditure in a modified price system, to attain greater satisfaction.

Now it is legitimate to conclude from this that $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} = P_0$ will be a ratio that, applied to a money-income R_0, would lead to a money-income R_1 which would always yield greater satisfaction than R_0. But it is wrong to say that $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = P_1$ is a ratio that, applied to a money-income R_0, leads to a money-income R_1 which will always yield less satisfaction than R_0. This will be true only if the amounts $\Sigma p_0 q_1$ and $\Sigma p_0 q_0$ are the same; for what is true of a money-income $\Sigma p_0 q_1$ (that is to say, that in situation 0 it yields more satisfaction than a money-income $\Sigma p_1 q_1$ in situation 1) is not necessarily true of a money-income $\Sigma p_0 q_0$ if it is different in amount.[2]

These observations of Konüs considerably limit Haberler's conclusions, in the sense that P_1 cannot be accepted as the lower limit of the variation except on the supplementary condition that

$$\Sigma p_0 q_1 = \Sigma p_0 q_0 \,{}^{3}.$$

This condition must therefore be added to those which Konüs elegantly describes as the conditions of "*konstanter Bedürfnisstand*" and "*zielbewusste Konsumtion*" (Bortkiewicz's translation)[1].

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[1] List of Sources, No. 2.

[2] If, for example, $\Sigma p_0 q_1 = 1{,}000$ and $\Sigma p_1 q_1 = 500$, it would be correct to say that in situation 0 a money-income of 1,000 represents a greater satisfaction-income than 500 in situation 1. But it does not not necessarily follow that by applying $P_1 = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = 0.5$ to an income $\Sigma p_0 q_0$ 1,000, say 500, the result would be an income which in situation 1 would procure less satisfaction than that derived in situation 0 from a money-income $\Sigma p_0 q_0 = 500$; for nothing is known of the relative satisfaction attaching to incomes other than $\Sigma p_0 q_1 = 1{,}000$ and $\Sigma p_1 q_1 = 500$, which relative satisfaction may be quite different.

[3] It is interesting to note that this condition comes to the same thing as an assumption that the index of the quantities is equal to unity, since we may write $\frac{\Sigma p_0 q_1}{\Sigma p_0 q_0} = 1$.

It is precisely to complete Konüs's reasoning that Bortkiewicz[2] considers another special case in which Haberler's conclusion might be maintained, namely, when it is known in advance that the two satisfaction-incomes corresponding to the two money-incomes R_0 and R_1 are equal. In these circumstances the argument may be as follows:

The two material-incomes $(q_0', q_0'', q_0''' \ldots)$ and $(q_1', q_1'', q_1''' \ldots)$ being by hypothesis equivalent in the satisfaction they yield, the money-income $\Sigma p_0 q_0 = R_0$ will necessarily be smaller than or at most equal to the money-income $\Sigma p_0 q_1$. For if $\Sigma p_0 q_0$ were greater than $\Sigma p_0 q_1$, the individual would not have consumed the material-income $(q_0', q_0'', q_0''' \ldots)$ but would have preferred to consume the material-income $(q_1', q_1'', q_1''' \ldots)$, which by hypothesis would have given him the same satisfaction-income at lower cost. This gives

$$(3) \qquad \Sigma p_0 q_0 \leqq \Sigma p_0 q_1 .$$

Similarly, we obtain

$$(4) \qquad \Sigma p_1 q_1 \leqq \Sigma p_1 q_0 .$$

If I denotes the ratio between the equivalent money-incomes R_1 and R_0, that is to say

$$I = \frac{R_1}{R_0} ,$$

a simple transformation of (3) will give

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \geqq \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} ; \quad \text{that is } I \geqq P_1 ,$$

and a transformation of (4)

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} \leqq \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} ; \quad \text{that is } I \leqq P_0 .$$

It should be observed that Bortkiewicz admits that in the extreme cases I may reach one or other " limit ". This hardly seems correct. For it is difficult to see why an individual should choose the material-income $(q_0', q_0'', q_0''' \ldots)$ rather than the equivalent material-income $(q_1', q_1'', q_1''' \ldots)$ if the cost of the two is identical in both situations.

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[1] In an article: " Der volkswirtschaftliche Geldwert and die Preisindexziffern, Eine Erwiderung " (Weltwirtschaftliches Archiv, vol. 30 (1929 II) pp. 6**-14**) Haberler has accepted Bortkiewicz's criticism (List of Sources, No. 2) and Konüs's argument. But he does not attach great importance to the resulting limitations on his theory, since, as a rule, comparison is confined to the cost of living of persons belonging to the same social class.

[2] List of Sources, No. 6.

For this reason we prefer to limit the conclusion of his reasoning to the formula

$$P_1 < I < P_0 .$$

* * *

This same case, where the equivalence of the two money-incomes is known in advance, was studied by Keynes[1] in 1930 (theory known to Bortkiewicz), in the following manner:

Assume that the material-income (q_0', q_0'', q_0''' ...) is measured in units representing the quantity of this collection of commodities that can be obtained for a unit of money, for instance £1, in situation 0; and that a similar unit is used to measure the quantity represented by the material-income (q_1', q_1'', q_1''' ...) in situation 1. Further, let $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}$. £1 be the price of this unit of (q_0', q_0'', q_0''' ...) in situation 1, and $\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}$. £1 the price of the unit of (q_1', q_1'', q_1''' ...) in situation 0. Finally, let $n_0 = \Sigma p_0 q_0$ be the number of units of (q_0', q_0'', q_0''' ...) bought in situation 0 by persons with a given satisfaction-income, and $n_1 = \Sigma p_1 q_1$ be the number of units of (q_1', q_1'', q_1''' ...) bought in situation 1 by " similar " persons with the same satisfaction-income.

In these conditions, the index of the cost of living will be the ratio between the money-incomes, or $\frac{\pounds n_1}{\pounds n_0} = \frac{\pounds \Sigma p_1 q_1}{\pounds \Sigma p_0 q_0}$, which will always lie between

$$P_0 = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \quad \text{and} \quad P_1 = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} .$$

For in situation 0 the individual can choose between consuming $n_0 = \Sigma p_0 q_0$ units of q_0's and

$$\pounds\, \Sigma p_0 q_0 : \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \cdot \pounds 1 = \Sigma p_0 q_0 \cdot \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \quad \text{units of } q_1\text{'s} ;$$

but since he prefers the q_0's and the satisfaction-income corresponding to this material-income is by hypothesis the same as that corresponding to $n_1 = \Sigma p_1 q_1$ units of q_1's, it necessarily follows that

$$\Sigma p_1 q_1 > \Sigma p_0 q_0 \cdot \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} ;$$

that is,

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} . \qquad \text{Therefore } \frac{R_1}{R_0} > P_1 .$$

Further, in situation 1, the individual can choose between $n_1 = \Sigma p_1 q_1$ units of q_1's and

$$\pounds\, \Sigma p_1 q_1 : \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} . \pounds 1 = \Sigma p_1 q_1 \cdot \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0} \quad \text{units of } q_0\text{'s} ;$$

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[1] List of Sources, No. 5.

but since he prefers the q_1's, and the satisfaction-income corresponding to the material-income composed by these q_1's is by hypothesis the same as that corresponding to $n_0 = \Sigma p_0 q_0$ units of q_0's, it necessarily follows that

$$\Sigma p_0 q_0 > \Sigma p_1 q_1 \cdot \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0} ;$$

that is,

$$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0} < \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} . \qquad \text{Therefore } \frac{R_1}{R_0} < P_0 .$$

To sum up, therefore,

$$P_1 < \frac{R_1}{R_0} < P_0 .$$

In order to avoid new signs, the whole of Keynes's reasoning has been converted to our notation. With his own notation his exposition is naturally less cumbersome and more elegant. However, if Bortkiewicz had published his argument reproduced above before Keynes's book appeared, the latter would perhaps have refrained from saying that his formula was simpler than others (he mentions Pigou and Haberler) and that his proof was more rigorous.

The theories so far presented have one important aspect in common. They lead to the conclusion of the superiority of one satisfaction-income over another according to the choice (fictitious, by the way) of an individual whose tastes are assumed to be the same in the two situations considered. The differences between the various writers lie chiefly in their manner of constructing the situation in which the fact of a certain consumption may be interpreted as a choice.

We now turn to two other theories, the first of which reasons directly in terms of satisfactions, and the second, while basing its argument on choices, makes use of a special instrument for representing these, namely " curves of indifference. "

In his article *Notes on Index-Numbers* [1], Bowley raises the question: what modification of total expenditure would be necessary to obtain the same total satisfaction in two situations. For this purpose he assumes that the expression for the dependence of the total satisfaction (or satisfaction incomes) of the average man on the quantities of each article is given and constant during the period of observation. Let this expression be as follows:

$$(5) \qquad U(x_1, x_2, x_3 \ldots x_n) ,$$

the x's representing the quantities of each article and U the resulting satisfaction-income.

In the first situation, the values of x_1, x_2, x_3 ... which make U a maximum, given the prices p_0', p_0'', p_0''' ... and the income $R_0 = x_1 p_0' + x_2 p_0'' + x_3 p_0''' + \ldots$, will be determined by the conditions:

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[1] List of Sources, No. 4.

$$\frac{\partial U}{\partial x_1} = Mp_0' \; ; \quad \frac{\partial U}{\partial x_2} = Mp_0'' \; ; \quad \text{etc.}$$

that is,

$$M = \frac{1}{p_0'} \cdot \frac{\partial U}{\partial x_1} = \frac{1}{p_0''} \cdot \frac{\partial U}{\partial x_2} = \quad \text{etc.}$$

The values of x_1, x_2, x_3 ... satisfying these conditions will be the same as q_0', q_0'', q_0''' ... if it is assumed that the individual, having a given money-income and faced with a given price system, tries to make U (the satisfaction-income) as large as possible. M represents the marginal utility of money for the average man in question.

Similarly, in the second situation[1]:

$$m = \frac{1}{p'_1} \cdot \frac{\partial U}{\partial x_1} = \frac{1}{p_1''} \cdot \frac{\partial U}{\partial x_2} = \quad \text{etc.}$$

where the values of x_1, x_2, x_3 ... satisfying these conditions are the same as q_1', q_1'', q_1''' ..., and m is the marginal utility of money for the same average man.

Having laid down these conditions, Bowley proceeds to determine the value of the difference between $U(q_0)$ and $U(q_1)$, that is, between the two satisfaction-incomes (which are of course not supposed to be equal, but are simply taken such as they are for the average man in the two situations, so that they are probably different). After a long series of transformations involving the use of Taylor's theorem—an instrument fancied by Cournot—which it is unnecessary to reproduce here, Bowley arrives at the following:

$$(6) \qquad U(q_1) - U(q_0) = \tfrac{1}{2}\Sigma(Mp_0 + mp_1) \cdot (q_1 - q_0) \; .$$

This is a first expression for the difference between the two satisfaction-incomes.

A supplementary hypothesis is now introduced, namely, that the relatively small variations of the quantities q_1', q_1'', q_1''' ... ,

$$\text{e.g. } q_1' + \nu_1 , \quad q_1'' + \nu_2 , \quad q_1''' + \nu_3 , \ldots$$

would be sufficient to make the satisfaction-income in the second situation $U(q_1)$ equal to that in the first situation $U(q_0)$. It is obvious that this new hypothesis imposes fairly important restrictions on the possibility of applying the theory. We shall return to this point later. This hypothesis may be given in the following form:

$$(7) \qquad U(q_1 + \nu) = U(q_0) \; ,$$

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[1] These conditions obviously imply R_1, the money-income of the individual in the second situation.

which, by a further application of Taylor's theorem—neglecting certain quantities which may be assumed to be small, taking (6) into account, and denoting the ratio of the marginal utilities of money by $\mu = \frac{M}{m}$ —, leads to

$$(8) \qquad \Sigma v p_1 = - \tfrac{1}{2}\Sigma(\mu p_0 + p_1) \cdot (q_1 - q_0) \; .$$

This is a second expression for the difference between the two satisfaction-incomes.

By hypothesis

$$I = \frac{\Sigma(q_1 + v) p_1}{\Sigma q_0 p_0}$$

for the ratio of equivalent incomes. Using (8), we have

$$I = \frac{\Sigma q_1 p_1 + \Sigma v p_1}{\Sigma q_0 p_0} = \frac{\Sigma q_1 p_1 - \tfrac{1}{2}\Sigma(\mu p_0 + p_1) \cdot (q_1 - q_0)}{\Sigma q_0 p_0} \; .$$

After a further series of transformations, and neglecting among other things the product of the difference between I and μ and another small quantity, we arrive at the conclusion that the value of I may be given by

$$(9) \qquad I = \frac{\Sigma(q_0 + q_1) p_1}{\Sigma(q_0 + q_1) p_0} = P_2 \; .$$

By a proof, also due to Bowley, given on page 61, note 1, it will be remembered that

$$P_0 \gtrless P_2 \gtrless P_1 \; .$$

Bowley's reasoning summarised above thus provides a further and entirely independent check of the theorem that the true index of the cost of living must lie between P_0 and P_1. It should also be observed that the initial hypotheses limit the application of this theory to the case where the tastes of the average individual remain unchanged. By the hypothesis introduced by (7) a further important limitation is created; as a matter of fact it was already implicit in the first application of Taylor's theorem—namely, that the satisfaction-incomes must not be too different in the two situations. This condition was one observed already in connection with Haberler's theory.

We now turn to the most recent, and, in our opinion, the most elegant of all the theories under consideration here, that of R. G. D. Allen [1], a writer who did not become acquainted with Haberler's book until after he had developed the results to be described [2].

Allen distinguishes between two problems. The first is to know what may be said of the total expenditure that a given individual should incur if, in any situation (even hypothetical), where the only change relates to prices, he wishes to obtain the same total satisfaction as in a given situation for which both his money-income and all prices are known. The second problem is to discover in two given situations, differing only in respect of prices, in which the individual has equivalent money-incomes, the means of calculating the theoretical "price of living." The same method is used to solve both problems.

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[1] List of Sources, No. 7.

[2] Communication contained in a letter from Mr. Allen.

In order to allow of graphical representation in two dimensions, it is assumed that there are only two commodities on the market. In this case the consumer's equilibrium is defined by the two conditions (using Bowley's notation):

$$\frac{1}{p_0'} \cdot \frac{\partial U}{\partial x_1} = \frac{1}{p''_0} \cdot \frac{\partial U}{\partial x_2} \tag{10}$$

and

$$x_1 p_0' + x_2 p_0'' = R_0 \ . \tag{11}$$

The values of x_1 and x_2 satisfying these conditions will be the quantities q_0' and q_0''.

Now, this equilibrium position may be represented geometrically by means of indifference curves, which can be defined as follows: If in a system of two co-ordinates the quantities of a first commodity are measured along the axis of abscissae and those of a second on the axis of ordinates, then any point in the first quadrant will represent a combination of quantities of these two commodities. An indifference curve will be a curve joining the points representing the combinations between which the choice of a given individual who is guided solely by his tastes will be indifferent, or in other words, which procure for a given individual the same total satisfaction. Such a line can be compared to a "curve of equal levels" as used for geographical maps; in fact it represents the level of total satisfaction. It is clearly necessary, to maintain this level, that any diminution in one of the commodities must be compensated by an increase in the other. It follows that each curve of indifference steadily falls from left to right. But the increase in commodity x_1 necessary to compensate the same diminution in commodity x_2 will not be the same along the whole of the indifference curve. It must increase according as the quantity of commodity x_2 still in the possession of the individual diminishes, and the quantity of commodity x_1 already in his possession increases. Each indifference curve will therefore be convex to the origin of the system. The whole of the first quadrant can be covered by indifference curves, none of which cut one another, and this will give a complete picture of the tastes of the individual in question [1].

By means of this system of indifference curves, equations (10) and (11) can be solved geometrically. For if x_1 is taken as the abscissa and x_2 as the ordinate, equation (11) may be written as follows:

$$x_2 = \frac{R_0}{p_0''} - \frac{p_0'}{p_0''} \cdot x_1 \ . \tag{11a}$$

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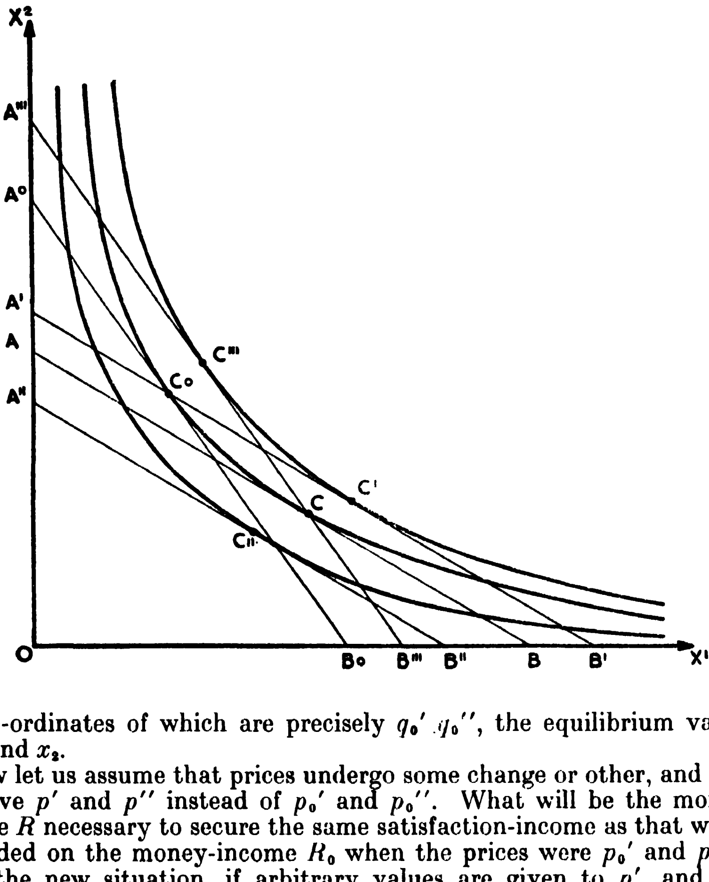
[1] For more detailed consideration of the properties of these curves, cf. Vilfredo Pareto, *Manuel d'Economie politique*, 2nd edition, Paris, 1927, pp. 168-175, 263-272, 572-579.

In the diagram, it is therefore represented by a straight line, e.g. A_0B_0. Equation (10) expresses that the total satisfaction of the individual reaches a maximum. Considered graphically, this means that when faced with the prices and income in the situation represented by the straight line A_0B_0, the individual will choose among all the possible combinations of x_1 and x_2 that which brings him to the indifference curve corresponding to the highest level of satisfaction his situation allows him to reach; in other words, he will take up the position where the straight line A_0B_0 is tangent to an indifference curve. This will be at the point C_0,



the co-ordinates of which are precisely q_0' q_0'', the equilibrium values of x_1 and x_2.

Now let us assume that prices undergo some change or other, and that we have p' and p'' instead of p_0' and p_0''. What will be the money-income R necessary to secure the same satisfaction-income as that which depended on the money-income R_0 when the prices were p_0' and p_0'' ? In the new situation, if arbitrary values are given to p', and p'', there will be parallel straight lines, with the equation

$$(11b) \qquad x_2 = \frac{R}{p''} - \frac{p'}{p''} \cdot x_1$$

to a number equal to that of the possible values of R, e.g. AB, $A'B'$, $A''B''$. One of these straight lines, say AB, will touch the same indifference curve as A_0B_0, but of course not at the point C_0, but at C. If the individual chooses the combination C in the new situation, he will therefore obtain the same satisfaction as from the combination C_0 in the base situation. Consequently, the particular value of R corresponding to this straight line AB is the money-income sought and the ratio $I = \frac{R}{R_0}$ is the theoretical " price of living ".

Let us now consider among the straight lines with equation (11*b*) that to which the value of R representing the cost of the old quantities q_0' and q_0'' at the new prices p' and p'' corresponds. It will be that which passes through the point C_0, that is to say $A'B'$ in the diagram. It calls for an expenditure $R' = p'q_0' + p''q_0''$. It will be seen that if the individual had this income R', he would spend it otherwise, for he would choose the position C', where $A'B'$ touches an indifference curve higher than (that is, to the right of) that touched both by A_0B_0 and by AB. The indifference curves being convex, the expenditure R' will thus be higher than that necessitated by one or other of these two straight lines, and in this case

$$R < R' = p'q_0' + p''q_0'' ,$$

from which it may be deduced that

$$\frac{R}{R_0} < \frac{R'}{R_0} = \frac{p'q_0' + p''q_0''}{p_0'q_0' + p_0''q_0''} .$$

Since the reasoning is equally valid for any number of articles, and since $I = \frac{R}{R_0}$, we may generalise, obtaining

$$I < \frac{\Sigma p_i q_0}{\Sigma p_0 q_0} = P_0 ; \qquad (i = 1, 2, \ldots).$$

The change in money-income needed by a given individual to obtain in any other price situation whatever a total satisfaction equal to that derived from a given money-income in the first price situation is therefore always lower than that indicated by P_0.

Concerning the second question treated by Allen, it is the same as that studied by Bortkiewicz and Keynes. It presupposes a knowledge not only of the prices and money-income in the base situation (the knowledge of any prices determining the second situation is needed only for the first problem), but also the actual prices and the money-income procuring for the individual in question the same satisfaction-income in a second given situation. In this second situation it is obvious that the general theory set forth above applies equally. We therefore have $I = \frac{R_1}{R_0} < P_0$, where R_1 is the money-income in the second situation, by hypothesis equivalent in satisfaction to the money-income R_0. *Mutatis mutandis* by writing p_1' and p_1'' for p_0' and p_0'' in equations (10) and (11), we can determine the values of x_1 and x_2 in the second situation, namely, q_1' and q_1''. They will be the co-ordinates of point C. The straight line AB will now have the equation

$$(11c) \qquad x_2 = \frac{R_1}{p_1''} - \frac{p_1'}{p_1''} \cdot x_1 .$$

The cost of the quantities q_1' and q_1'' at the prices of the first situation will be $p_0'q_1' + p_0''q_1''$, represented by the straight line $A'''B'''$. If the individual had this money-income, which we may call R''', he would spend it in the manner C''', and would thus obtain a satisfaction higher than that represented by the combination C. To obtain a satisfaction equal to C, he would in situation 0, where the prices are p_0' and p_0', arrange his expenditure according to the combination C_0, which is based on the assumption of a money-income R_0. Owing to the convexity of the indifference curves this means that

$$R''' > R_0 \,,$$

that is, $$p_0'q_1' + p_0''q_1'' > p_0'q_0' + p_0''q_0'' \,.$$

Therefore
$$\frac{p_0'q_1' + p_0''q_1''}{p_1'q_1' + p_1''q_1''} > \frac{p_0'q_0' + p_0''q_0''}{p_1'q_1' + p_1''q_1''}$$

and
$$I = \frac{R_1}{R_0} > \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} = P_1 \,.$$

* * *

Before trying to draw any general conclusions from the theories briefly set forth above, it seems advisable, not to compare the hypotheses made by the different writers—since in essence these are the same—but to bring out the concrete circumstances in which each writer seems inclined to think that his hypotheses will be realised.

For Bowley, for instance, the assumption that the form and constants of the function U are unchanged in the two situations " limits the measurement to an interval of time in which customs and desires have not changed and to a not very wide range of real income. The analysis and conclusions do not apply to comparisons between citizens of two countries, nor over, say, 60 years in one country. The relevance can be tested by general statistics of consumption, which show for England that the changes in habit were slight between 1904 and 1927 " (pp. 223-224)[1]. Keynes approves this opinion of Bowley's (p. 120). Pigou considers that " the assumptions, upon which the mathematical defence of existing index numbers . . . rests, are, perhaps, less fully realised in fact than theoretical statisticians are sometimes willing to concede " (p. 40). He considers, however, that satisfactory results can be obtained from his own method if the comparison relates (in one country) to closely adjacent times (p. 50), whereas it must fail when applied to distant times (p. 51). Haberler says that even over a small number of years exact results cannot be expected (p. 88). As regards Konüs (and with him Bortkiewicz), his hypothesis is known; he gives no indication as to the circumstances in which it may be expected to be realised.

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[1] The references are to the works mentioned in the List of Sources given at the end of this appendix.

It may thus be observed that these writers, whether explicitly or not, deny the possibility of comparisons in space. They agree, however, that comparisons can be made between two distinct periods, that is to say, separated by a more or less short but effective interval.

In this respect Allen adopts a more rigorous point of view. He says in effect (p. 201): The theoretical notion of the price of living " refers necessarily to one individual only, . . . faced with various price situations on a given consumers' goods market. The concept is in no way a collective one, nor can it refer to the same, or different, individuals at different times or on different markets. In particular, the consumers' goods are a given and invariable set ". Elsewhere he adds (p. 205): The " price of living " depends among other things on the individual's level of expenditure. . . . The theory given here refers only to one individual with given tastes, and the significance of the fact that his income is assumed to be known is merely that consideration must be confined to one particular income among all those possible. The different price situations are similarly altogether hypothetical. The results therefore merely show what would be the " price of living " for a given individual—whose income is assumed to be known—if prices at a given moment, instead of being what they are, were different[1]. He concludes by saying (p. 208) that the problem of the " geographical " price of living is incapable of theoretical solution. There is no theoretical support for a comparison of the price of living in two countries or markets with different modes of living. Any such comparison must necessarily be rough and empirical.

Two conclusions may be drawn from the theories described above. The *first* is that the indexes constructed by means of the formulae P_0 and P_1 represent for each of the writers studied the upper and lower limits of the theoretical price of living if the comparison relates to an individual (actual or average) who is faced with identical conditions as to satisfaction-income and tastes in two situations which differ only in respect of prices. As to the supplementary condition formulated by Konüs, namely that $\Sigma p_0 q_0 = \Sigma p_0 q_1$, we agree with Bortkiewicz that it may be substituted for the *a priori* knowledge of equality of satisfaction-incomes[2]. Further, Konüs's objection that Haberler's theorem is too general in form appears to us entirely justified. It is merely another aspect of the observation made on page 77 above in the discussion of Haberler's ideas[3]. It may be added that Allen would agree with this point of view, as appears clearly from the distinction he makes between the two problems that form the subject of his study. Like Konüs, in the examination of his first problem, he comes to the conclusion that only P_0 forms a limit (the upper limit) to the theoretical price of living when the equality of satisfaction-incomes in the two situations is not known in advance. He, too, considers it necessary to know the amount of money-income which in a second given price situation would provide a given individual with the same satisfaction-income before reaching any conclusion as to the nature of the lower limit for P_1.[1] Haberler himself claims no validity for his conclusions in the case of too large a difference between the satisfaction-incomes in question. It will be remembered that, as mentioned above, he has accepted Konüs's theory.

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[1] We have taken the liberty of somewhat modifying Allen's expressions.

[2] To check whether this is true, use cannot be made of Allen's graphic representations, since they can take only two commodities into account, and the straight lines representing the two sides of the above equation therefore coincide.

[3] It would even be possible to accuse Haberler of a theoretical error. He states nowhere that it is necessary to know in advance the income R_1 which, in the second price situation, is equivalent to R_0 if P_1 is to represent the lower limit of the price of living. He even admits the possibility that the theoretical price of living may be either higher than P_0 or lower than P_1 (p. 94), which is impossible unless the hypotheses on which the whole of his reasoning rests are not realised. To put the case in a more practical form, the situation may be formulated as follows:

As only P_0 can be calculated by means of the quantities of the initial period — given that P_1 involves the quantities consumed in the second period— it is fairly plausible that no precise statement can be made as to the relation between P_1 and the theoretical price of living when the satisfaction-income procured from the material-income q_1, is unknown.

All the writers in question are unanimous on a *second* point (and their unanimity is all the clearer in that some of them do not even trouble to mention it). This is that it would be useles to try to determine the price of living as between two countries.

In the theory formulated in Chapter II of the first part of this volume we adopted as our starting point a definition of the cost of living which involved the equivalence of money-incomes in the two situations to be compared. But research was not directed towards discovering these equivalent incomes. On the contrary, criteria were developed to allow of picking out from the comparisons of a series of incomes in each of the two countries in question that which was more acceptable than the rest, and only after this " best " comparison had been determined was it found that it combined incomes which—according to altogether independent criteria—might be considered equivalent. It may therefore be noted that, without completely departing from what constitutes the first point established by the theories described above—and, in particular, without directly contradicting it—we have not used it as the guiding line for our research.

Further, the very problem we have set ourselves is in flagrant contradiction with all these theories, according to which no rigorous comparison is possible in space, since there is no identity of tastes or of consumption habits between two localities.

In these circumstances it is interesting to note that the results of our analysis are a confirmation of these theories. For our search, intended to select from among two sets of incomes (relating to two different countries) those two between which comparability would be as great as possible, has led us to select combinations of incomes which, on the basis of an entirely independent theory—that of the " valley of equivalence "—may be considered as approximately equivalent, or in any case as equivalent as possible. This, it should be noted—notwithstanding all the objections that can be raised against the " empirical " nature of deductions based on actual data—is not only a further confirmation of these writers' theories. It is at the same time an indication that the theory of the " valley of equivalence " cannot be absolutely wrong. It may be added that the reason why all these writers regard inter-local comparisons as impossible is essentially that there is no identity of tastes between two countries; and such identity of taste is an indispensable condition—irrespective of any identity of satisfaction-incomes—for the conclusion that P_0 and P_1 limit the ideal solution. It is certain that this is so in pure theory; either the identity of tastes and customs is complete, or we must give up any idea of a rigorous conclusion, even though it can be supposed that the two incomes in question are equivalent. Is this a reason, however, to refrain from attempting any inter-local comparison ? Would it not be justifiable to admit the existence of intermediate states between the identity and the non-identity of tastes ? In other words, it seems to us altogether possible to admit that the tastes of different individuals may be more or less similar, and to conclude from this that the more they resemble each other, the more will the values of P_0 and P_1—assuming the equivalence of money-incomes to be given -approach the true limits [1]. This must not be taken to mean that pure theories are of no use; on the contrary, it is only by the consideration of pure cases that rigorous conclusions can be reached. But once these conclusions have been reached it is necessary by means of an appropriate adjustment of the hypotheses to seek for a reasonable method of approximation. Here, we believe, lies the essence of the apparent contradiction between the points of view of the above-mentioned writers and ourselves. We readily admit that the practical results we have arrived at are not as rigorous as the theoretical conclusions of these writers; but what we have lost in theoretical strictness we have gained in practicability.

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[1] In a letter Mr. Allen has defined his position as follows: " $I = \frac{R_1}{R_0}$ depends on one or other of the two equivalent incomes; $I < P_0$, if the first equivalent income (R_0) is the one we start from, $I > P_1$, if the second. $P_1 < I < P_0$ only if both equivalent incomes are known."

In conclusion, we wish to draw attention to a serious omission in the whole of our survey. The theories studied, like our own, which is set forth in Chapter II of the first part of this volume, relate to the cost of living as a whole, whereas our quantitative treatment relates only to foodstuffs [2]. How far can food indexes be accepted when their theoretical justification is effected in terms of the cost of living ? The consideration of this problem would call for research into the question of " consumers' equilibrium ", a difficult but indispensable task if it is proposed to extend the scope of these international comparisons.

This task will be reserved for future research, like so many other questions that could be no more than raised during the course of this study.

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[1] See pages 12, 13 above. The approach of P_0 on the one hand and P_1 on the other to the " true limits " must, of course, be distinguished from an approach between the values of P_0 and P_1 themselves. As pointed out on page 14 above, the values of P_0 and P_1 may be very close to each other without either of them being close to the " true limits ".

[2] This applies also to the " valley of equivalence "; but here it may be shown that generalisation to the equivalence of the whole income is legitimate, although the statistical researches take only foodstuffs into account.

LIST OF SOURCES

N.B. — This list, which does not claim to be a " bibliography ", merely enumerates the works discussed in the above appendix. A complete bibliography of the question of the measurement or comparison of standards of living would obviously not omit such a work as *New Methods of Measuring Marginal Utility*, by Ragnar Frisch.

Further, it is thought desirable to utter a warning—superfluous to statisticians, but perhaps not altogether valueless for economists—against confusing the problems connected with the ideas of the cost and standard of living with the theory of index numbers of prices. It is in fact possible to imagine a theory of cost of living comparisons which not only does not make direct use of index numbers of prices—as is the case in Frisch's book—but is moreover entirely non-mathematical. Further, the theory of index numbers, far from serving only for the comparison of costs of living, is properly speaking no theory, but a body of technical considerations and indications which can lead to the conclusion that certain formulae are superior to others, and even (as in Irving Fisher's *Making of Index Numbers*) that the formulae of Laspeyres and Paasche are to be specially recommended, whether taken concurrently or in the form of some average. But the reasons put forward in the theory of the cost of living in favour of these formulae must not be taken as a justification of the same order of value. It is mere chance that Fisher's " ideal " formula is the geometric mean of the formulae which, in the theory of the cost of living, play the important part described above.

1. Pigou, A. C.: *Wealth and Welfare*, London (Macmillan), 1912, Chapter III: " The Measurement of the Dividend and its Parts ", pp. 33-51.

See also the first three editions of *The Economics of Welfare*, 1920, 1924 and 1929. The fourth was not available.

2. Konüs, A. A.: *Economic Bulletin*, organ of the Moscow Institute of Economic Research, 1924, Nos. 9-10, pp. 64-71 (in Russian).

Cited by L. v. Bortkiewicz in his account of G. Haberler's book (see No. 3 below): *Magazin der Wirtschaft*, Berlin, Vol. 4 (1928), No. 11, pp. 427-429.

3. Haberler, Gottfried: *Der Sinn der Indexzahlen*, Tübingen (Mohr), 1927, Part B, Chapter II: " *Der Einkommensindex* ", pp. 77-99, and in particular pp. 81-98.

See also " *Der volkwirtschaftliche Geldwert und die Preisindexziffern eine Erwiderung* ", in Weltwirtschaftliches Archiv, vol. 30 (1929 II), pp. 6**-14**.

4. Bowley, A. L.: " *Notes on Index Numbers.* " *The Economic Journal*, Vol. XXXVIII (1928), pp. 216-237, and in particular pp. 223-229.

5. KEYNES, John Maynard: *A Treatise on Money*, London (Macmillan), 1930, Vol. I, Book II, Chapter 8: " The Theory of Comparisons of Purchasing Power ", pp. 95-120, and in particular pp. 109-111.

6. BORTKIEWICZ, L. von: "*Die Kaufkraft des Geldes und ihre Messung.*" *Nordic Statistical Journal*, Vol. 4 (1932), pp. 1-68, and in particular pp. 17-23.

In this work Bortkiewicz again reproduces the theory of Konüs.

7. ALLEN, R. G. D.: "*On the Marginal Utility of Money and its Application.*" *Economica*, No. 40 (May 1933), pp. 186-209, and in particular pp. 200-209.

APPENDIX II

DESCRIPTION AND SOURCES OF THE DATA USED

A. — DATA RELATING TO QUANTITIES OF FOODSTUFFS CONSUMED

Countries	Localities covered	Period of enquiry	Social class	Number of families	Average yearly income per consumption unit (adult male)	Sources
1. **Germany**	56 towns	March 1927-February 1928	Workers Civil servants	Minimum 102 Average 896[1] Maximum 78	RM. 700 ,, 1,200 ,, 3,400	**Statistisches Reichsamt:** *Wirtschaft und Statistik,* 1929, 2. Dez.—Heft, No. 24, and 1930, 2. April-Heft, No. 8.
2. **Austria**	Vienna	Year 1930	—	Average 72	Sch. 1,500[2]	**Kammer für Arbeiter und Angestellte:** *Wirtschaftsstatistisches Jahrbuch 1930-1931,* VII. Jahrgang. Vienna, 1932.
3. **Belgium**	Whole country[3]	4 fortnights in 1928-1929	Workers	Minimum 192 Average 412 Maximum 49	Frs 3,600 ,, 5,200 ,, 9,700	Communication from the Ministry of Industry and Labour to the International Labour Office.
4. **Bulgaria**	Whole country[3]	June 1927-May 1928	Workers	Average 93	Leva 14,800	**Central Statistical Office:** *Annuaire statistique du Royaume de Bulgarie.* Sofia, 1931.
5. **Denmark**	Copenhagen	Year 1922	Workers and civil servants	Minimum 16 Average 67 Maximum 15	Kr. 1,600 and less ,, 2,400 ,, 3,200 and more	**Department of Statistics:** *Husholdningsregnskaber for 1922.* 4. Raekke, 69. Bind, 5. Haefte.
6. **Estonia**	Tallinn	October 1925	Workers	Average 176	Kr. 400	**Central Office of Statistics:** *Eesti Töölise Büdzhet 1925* a. Tallinn.
7. **Finland**	I. 16 towns and 8 rural centres II. Helsinki	March 1920-February 1921	I. Workers, Salaried employees II. Workers	Minimum 92 Average 437 Maximum 83 Average 121	Mk. 4,500 ,, 6,300 ,, 14,900 ,, 7,000	**Central Bureau of Statistics:** " Sosialisia Erikoistutkimuksia ": *Coût de la vie pendant l'exercice annuel 1920-1921.* Helsinki, 1925.
8. **Latvia**	Riga	1926-1927	Workers	Average —	—	**Bureau of Statistics:** *Méthode nouvelle du calcul des nombres-indices du coût de la vie.* Riga, 1931.
9. **Norway**	Oslo	September 1927-August 1928	Workers	Minimum 12 Average 44 Maximum 2	Kr. 1,170 ,, 1,430 ,, 2,450	**Central Statistical Office:** *Husholdningsregnskap 1927-1928.* VIII, 103. Oslo, 1929.

DESCRIPTION AND SOURCES OF THE DATA USED *(continued)*

— DATA RELATING TO QUANTITIES OF FOODSTUFFS CONSUMED *(continued)*

Countries	Localities covered	Period of enquiry	Social class	Number of families	Average yearly income per consumption unit (adult male)	Sources
10. POLAND	I. Warsaw	I. (*a*) Year 1927	I. Workers 1927	Minimum 9	Zl. 500	CENTRAL OFFICE OF STATISTICS: *Budgets des familles ouvrière 1927, 1928, 1929, Statistique de la Pologne*, Tome XL, Fasc. 1 and 2. Warsaw, 1930 and 1933.
				Average 40	,, 800	
				Maximum 8	,, 1,500	
		(*b*) Year 1928	1928	Average 23	,, 900	
		(*c*) Year 1929	1929	Average 18	,, 900	
	II. Upper Silesia	II. Year 1927	II. Workers 1927	Average 44	,, 900	
11. SWEDEN	I. Stockholm	I. Year 1922-1923	I. Workers and Salaried employees	Minimum 39	Kr. 1,140	I. STOCKHOLMS STADS STATISTIK: *Levnadskostnaderna i Stockholm 1922-1923.* X. Special undersökningar, No. 13. Stockholm, 1927.
				Average 140[4]	,, 1,570	
				Maximum 36	,, 2,250	
	II. Göteborg	II and III. Year 1923	II. ,, ,,	Average 77[5]	,, 1,240	I. II. III. K. SOCIALSTYRELSEN: *Levnadkostnaderna i Städer och industriorter 1923.* Stockholm, 1929.
	III. Malmö		III. ,, ,,	Average 51[6]	,, 1,200	
12. SWITZERLAND	Winterthur	Year 1922	Workers	Average 46 } 56	Frs. 2,000 2,000	*Statistische Mitteilungen betreffend den Kanton Zürich*, Heft 150. *Haushaltungsrechnungen aus der Stadt Winterthur, 1921-1922.*
			Salaried employees	10	2,300	
13. CZECHOSLOVAKIA	I. Whole country	I. July 1927-June 1928	Workers I.	Minimum 12	Kc. 5,800	*Mitteilungen des Statistischen Staatsamtes der CSR.* I. Jahrgang XII, 1931. Nos. 161-164 and 225-291
				Average 79	,, 6,500	
				Maximum —	—	
	II. Prague	II. (*a*) July 1927-June 1928	Civil servants I.	Minimum —	—	II. (*a*) Jahrgang X, 1929, Nos. 156-159.
		(*b*) July 1928-June 1929		Average 122	,, 12,300	(*b*) ,, XII, 1931, Nos. 211-215.
		(*c*) July 1929-June 1930		Maximum 10	,, 20,400	(*c*) ,, XIII, 1932, ,, Nos. 158-159.
			II. Workers 1927-1928	Average 27	,, 7,140	
			1928-1929	Average 45	,, 7,600	
			1929-1930	Average 61	,, 8,100	

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[1] The classes labelled " average " include the families labelled " maximum " and " minimum " and generally also other families data for which have not been used.

[2] Average yearly income *per person.*

[3] The number of towns covered by the enquiry is not given.

[4] Includes 55 salaried employees' families.

[5] Includes 38 salaried employees' families.

[6] Includes 27 salaried employees' families.

B. — DATA RELATING TO RETAIL PRICES OF FOODSTUFFS

Countries	Localities covered	Period of enquiry	Sources [1]	Kind of data
1. GERMANY	Berlin	July 1930	STATISTICAL OFFICE OF THE CITY OF BERLIN: *Berliner Wirtschaftsberichte*, VII. Jahrgang. No. 2-3, 17 September 1930.	Prices collected in retail shops and on markets.
2. AUSTRIA	Vienna	July 1930	FEDERAL STATISTICAL OFFICE: *Statistische Nachrichten*. VIII. Jahrgang. No. 7, 27 July 1930.	Prices collected on municipal markets.
3. BELGIUM	Brussels	January 1931	Communication from the Ministry of Industry and Labour to the International Labour Office.	Prices ordinarily paid by the working classes.
4. BULGARIA	12 towns	July 1930	DIRECTION GÉNÉRALE DE LA STATISTIQUE DU ROYAUME: *Bulletin mensuel de statistique*, XIX^e année. No. 8, August 1930.	—
5. DENMARK	Copenhagen	July 1930	DEPARTMENT OF STATISTICS: *Statistiske Efterretninger*, 22. Aarg. No. 23, 13 August. 1930.	—
6. ESTONIA	Tallinn	July 1930	CENTRAL OFFICE OF STATISTICS: *Eesti Statistika*, July-August 1930.	Prices collected in retail shops and on municipal markets.
7. FINLAND	40 towns	July 1930	MINISTRY OF SOCIAL AFFAIRS: *Socialinen Aikakauskirja*. 1930, No. 9.	—
8. LATVIA	Riga	July 1930	BUREAU OF STATISTICS: *Menosa Bileteno*. Monthly Bulletin, September 1930, No. 9.	—
9. NORWAY	Whole country	July 1930	CENTRAL BUREAU OF STATISTICS: *Statistiske Meddelelser*. 1931, No. 1-2, pp. 40-41.	—
10. POLAND	Warsaw Katowice	July 1930	CENTRAL OFFICE OF STATISTICS: *Statystyka Cen* (Statistique des prix). Tome II, fasc. 3, 1930.	Prices collected in retail shops.
11. SWEDEN	Stockholm Göteborg Malmö	July 1930	K. SOCIALSTYRELSEN: *Sociala Meddelanden*, 1930, No. 10.	—
12. SWITZERLAND	34 communes	July 1930	FEDERAL ECONOMIC DEPARTMENT: *La Vie économique*. III^e année. Fasc. 3, August 1930.	Current prices.
13. CZECHOSLOVAKIA	Prague	July 1930	STATISTICAL OFFICE OF THE CITY OF PRAGUE: *Bulletin mensuel*. VI^e année. No. 7, July 1930.	
		July 1927-June 1928 July 1928-June 1929 July 1929-June 1930	Same source as for corresponding quantities.	Prices obtained by dividing annual expenditure by annual quantity consumed.

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[1] Except for Bulgaria, Finland, Latvia and Switzerland, the retail prices taken from the sources indicated have also been compared with those communicated to the International Labour Office in connection with its enquiries into wages and retail prices in certain towns.

APPENDIX III

DATA USED IN THE CALCULATIONS

Introductory Note

The quantities adopted for each of the localities considered have been taken from the various budget enquiries. Unless otherwise indicated, the prices are generally those for July 1930, and have been taken from the official publications of the competent statistical departments of the countries concerned. In certain cases, however, use has been made of the prices communicated to the Office for the purpose of its enquiry into wages and retail prices in various towns.

The details and sources of these data are given in Appendix II.

The method of selecting the most representative prices for the quantities concerned has been, in the first place, to compare the prices actually paid at the time of the various budget enquiries for the articles consumed with the published retail prices for these same articles at the same time. The quality of articles whose retail prices varied not more than 10 per cent. has generally been taken as identical with that of the articles covered by the enquiries. Consequently, the prices taken for July 1930 have been those available for these qualities.

This is the case for most of the articles in nearly all the countries covered. Nevertheless certain modifications of the original data have been found necessary as regards both quantities and prices.

As a rule, these modifications affect the same articles in most of the countries, and accordingly it will suffice to explain below the principal adjustments made in the data. A special sign against these data in the table refers the reader to these explanations.

Certain isolated cases of adaptation of data are explained at the foot of the table.

(ǂ) For certain articles, the known quantity represents the consumption of two or more qualities, whereas the currently published prices of these various qualities differ but little from each other and from the average price actually paid for the total quantity. In order to arrive at an average price for this total quantity, a simple average has been taken of the retail prices recorded for the different qualities. When the difference between this average and the average price actually paid at the time of the budget enquiry did not exceed 10 per cent., a similar average calculated from the prices for July 1930 has been used in the tables.

(+) For certain articles, the classification in the budget enquiries in some countries is more detailed than in others. Hence it has proved necessary in some cases, where no other procedure seemed possible, to use the weighted average of the prices of two or more articles as corresponding to the sum of their quantities.

(*) For certain articles, the prices of which vary according to the quality (e.g. beef), it has been necessary to subdivide the quantity representing the total consumption of all the qualities into two or more items representing the quantities consumed of each quality.

The method adopted is the following:

Let q be the only quantity known,
p the price of the article concerned, actually paid at the time of the enquiries,
p^1 and p^2, the price of each quality of the article, taken from official publications and relating to the period covered by the enquiries,
x and y, the quantities to be obtained for each quality.

The price p is an average of the prices p^1 and p^2, weighted by the respective quantities of each quality. These quantities can therefore be obtained from the following equations:

$$\begin{aligned} x + y &= q \\ p^1x + p^2y &= pq \end{aligned}$$

Example:

The consumption of beef in Germany (medium income class):

Let 6.4 kg. be the total quantity known,
2.37 RM. be the price of 1 kg. of beef (enquiry of 1927-1928)
2.85 RM. the price of 1 kg. of roasting beef
2.25 RM. the price of 1 kg. of boiling beef
} average retail prices in Berlin recorded in 1927-1928

then

$$\begin{aligned} 2.37 \times 6.4 &= 15.17 \text{ RM.} \\ x + y &= 6.4 \text{ kg.} \\ 2.85\,x + 2.25\,y &= 15.17 \text{ RM.} \end{aligned}$$

hence

$$x = 1.3 \text{ kg.}; \quad y = 5.1 \text{ kg.}$$

N.B. — This calculation has been made from the data relating to the medium income class in the various countries. Once the consumption of various qualities thus obtained, the next operation was to calculate their ratio to the total quantity of the article concerned, and this ratio was then applied to all the other income classes in the table.

(x) Certain articles of which only the total quantities have been used, are composed of two or more items, the prices of which vary very slightly. Where the quantity of one of these items is predominant the price for this quality has been applied to the whole group.

(¶) For certain articles or groups of articles, the current retail prices are not published in such detail as the prices and quantities taken from the budget enquiries. In order to obtain a representative price for the articles whose prices are lacking, the procedure described below has been followed:

Calculation has been made of the ratio of (*a*) the average price (p^1) actually paid at the time of the budget enquiry for articles whose retail prices are not currently published, to (*b*) the simple average (p^2) of the prices of articles that may belong to the same group, these prices being the current retail prices recorded during the period of the enquiry. The coefficient $\left(\frac{p^1}{p^2}\right)$ was then applied to the average for July 1930 of the prices of the articles that have been used to obtain (p^2). The resulting estimated price is stated in the tables.

(†) In two countries, Austria and Bulgaria, the quantity given for the group " meat " represents the total consumption of all qualities of meat.

In order to estimate the quantities for each quality, use has been made of earlier budget enquiries published by the two countries in question, giving a specific subdivision for the group " meat ". A calculation has then been made of the proportion of each quality of meat in the total quantity for the group, and the coefficients thus obtained have been applied to the total quantity at the later date.

(§) Some countries indicate the consumption of milk and eggs in kg., and in this case use has been made of the following coefficients already adopted for other enquiries undertaken by the Office:

1 litre of milk = 1,031 grammes
1 kg. of eggs = 17.5 eggs.

DATA ON WHICH CALCULATIONS ARE BASED

Articles	GERMANY				AUSTRIA		BELGIUM				BULGARIA	
	Whole country			Berlin	Vienna		Whole country			Brussels	Whole country	
	Annual consumption per unit having an annual income of:			Prices in Berlin in July 1930	Annual consum. per head, having annual income of	Prices in Vienna, in July 1930	Annual consumption per unit having an annual income of:			Prices in Brussels in 1931	Annual consum. p. unit. annual income of:	Average price in July 1930
	RM. 700	RM. 1,200	RM. 3,400		Sch. 1,500		Fr. 3,600	Fr. 5,200	Fr. 9,700		Leva 14,800	
	workers		officials				workers				workers	
	Kg.	Kg.	Kg.	RM.	Kg.	Sch.	Kg.	Kg.	Kg.	Fr.	Kg.	Levas
1. Bread, white	14.8	20.2	26.4	0.813	28.09	1.59	202.97	195.41	182.09	1.74	0.00	7.94
2. Bread, rye or " black "	99.3	85.8	63.7	0.392	62.29	0.58					218.45	6.03
3. Flour, wheat	14.3	14.7	14.6	0.58	22.65	0.68		2.65		2.12	52.06	7.23 ¶
4. „ oat	—	—	—	—	—	—	—	—	—	—	—	—
5. „ rye	—	—	—	—	—	—	—	—	—	—	—	—
6. Butter, fresh or salted	2.7	5.9	15.8	3.37	5.16	6.00	9.74	17.35	22.05	24.57 ‡	0.53	67.50
7. Margarine	11.8	9.7	5.4	1.80	1.67	2.80	6.45	4.58	3.00	12.25	—	—
8. Lard	16.4	4.5	2.7	1.41	2.30	3.40	5.52	5.10	6.07	14.24	5.74	48.38
9. Beef, 1st quality	0.8 *	1.3 *	1.9 *	3.02	20.22 †	3.00 ‡	12.26	17.51	25.49	24.54 ‡	10.57 †	24.77
„ 2nd „	3.1 *	5.1 *	7.7 *	2.27								
10. Mutton	0.5	0.6	1.3	2.70	0.17 †	3.40	—	0.11	—	17.88 ‡	12.85 †	25.27
11. Pork	6.0	7.8	8.6	1.98	19.01 †	5.00	4.64	7.81	10.06	22.21 ‡	6.88 †	38.75
12. Veal	0.6	1.5	4.9	2.66	2.59 †	4.00	1.18	2.54	5.15	28.00 ‡	—	—
13. Sausage	9.4	13.4	14.5	2.83 ‡	11.29	3.80	4.22	4.20	6.09	26.83	—	—
14. Ham	0.2	0.7	2.3	5.01	—	—	0.80	1.28	2.27	27.58	—	—
15. Bacon	2.3	3.8	2.2	2.50	4.86	4.50	6.80	7.16	6.34	15.74	—	—
16. Potatoes	147.8	153.6	137.7	0.107	43.57	0.32	212.50	210.92	210.88	0.94	19.65	3.21
17. Sugar	14.1	16.6	18.7	0.60	23.71	0,94	11.78	14.45	17.51	2.44	9.60	30.42
18. Coffee	1.0	1.2	1.2	6.61	5.08	9.20	4.68	6.45	8.11	19.78	0.30	145.83
19. Tea	0.1	0.1	0.1	10.80	0.12	20.50	—	0.11	—	35.00	0.06	300.00
20. Cocoa	—	0.5	—	4.00	0.45	5.40	—	—	—	—	—	—
21. Cheese	3.6	4.8	6.3	1.87 ‡	3.14	5.40	3.63	5.00	6.99	21.50	6.01	26.46
22. Milk, unskimmed (litres)	109.4	150.1	172.6	0.31	166.99	0.54	114.68 §	134.54 §	154.21 §	2.20	26.69	7.92
23. Eggs (number)	78.0	152.0	232.0	0.12	240.00	0.14	111.65 §	187.78 §	258.65 §	1.03	71.93	1.60
24. Rice	—	1.9	—	0.70	4.21	0.90	—	4.81	—	5.21	5.09	19.04
25. Peas, dried	0.7 *	0.7 *	0.7 *	0.60	1.32	1.15 ‡	2.08	3.02	4.05	5.33 ‡	4.84	9.79
26. Beans, haricot	2.0 *	2.0 *	2.0 *	0.81								
27. Fish, fresh	3.1	3.1	4.9	0.79 ‡	—	—	—	—	—	—	—	—
28. „ other	3.1	3.5	3.6	1.23 ‡	—	—	—	—	—	—	—	—
29. Prunes	—	1.2	—	1.25	—	—	—	—	—	—	—	—

Germany: 13. Average price of 6 different qualities.
21. „ „ „ 2 „ „
27,28. Average price of 3 different qualities.

Austria: 9, 25, 26. Average price of 2 different qualities.

Belgium: 6, 9, 10, 11, 12, 25, 26. Average price of 2 different qualities.
3, 10-19, 24. Quantities and prices used for comparing district baskets. The data refer to the city of Antwerp only and were used for "Contribution ".
13 and 14. Prices for October 1930, used for "Contribution ". In order to make them apply to January 1931, use has been made of the coefficient of reduction 0.935.

Bulgaria: 3. Flour: Price obtained by calculating the ratio of the price at the enquiry to the price of bread, this ratio being subsequently applied to the price of bread in July 1930.

DATA ON WHICH CALCULATIONS ARE BASED *(cont.)*

Articles	DENMARK				ESTONIA		FINLAND					LATVIA	
	Copenhagen				Tallinn		Whole country			Helsinki	Whole country	Riga	
	Annual consumption per unit, having an annual income of:			Price in Copenhagen in July 1930	Ann. consum. p. unit, ann. income of:	Prices in Tallinn in July 1930	Annual consumption per unit, having an annual income of:			Average quantities	Average price in July 1930	Ann. consum. p. unit	Prices in Riga in July 1930
	Kr. 1,600	Kr. 2,400	Kr. 3,200		Kr. 400		Mcs. 4,500	Mcs. 6,300	Mcs. 14,900	Mcs. 7,000		—	
	workers and officials				workers		workers			workers		workers	
	Kg.	Kg.	Kg.	Kr.	Kg.	Kr.	Kg.	Kg.	Kg.	Kg.	Mcs.	Kg.	Lats
1. Bread, white	16.06	20.49	26.16	0.77	48.8	0.50	8.3	12.7	23.7	35.5	7.26 ¶	46.5	0.61
2. Bread, rye or " black "	84.77	84.48	73.03	0.17	173.3	0.14	29.0	33.7	31.3	51.0	4.04 ¶	96.3	0.19
3. Flour, wheat	14.77	15.68	12.53	0.32	9.2	0.50	23.1	29.7	27.3	33.7	4.75‡	12.6	0.55
4. „ oat	—	4.00	—	0.60	7.8	0.35	—	—	—	12.9	4.41	7.5	0.40
5. „ rye	—	—	—	—	0.6	0.104	99.3	81.8	35.9	32.6	3.29 ‡	—	—
6. Butter, fresh or salted	3.20	9.28	13.91	2.97	5.6	2.05‡	16.2	19.6	20.4	18.8	26.37 x	10.4	3.00
7. Margarine	19.78	15.69	13.41	1.46	3.1	1.38	2.0	2.1	4.6	2.4	17.86	—	—
8. Lard	3.39	3.17	1.91	1.95	1.8	1.53	1.7	1.5	2.1	1.3	16.29	2.2	2.22
9. Beef, 1st quality	1.73 *	2.22 *	1.95 *	2.83	2.3 *	1.20	12.5	16.5	25.9	18.7	11.55 ‡	4.7	1.40
„ 2nd „	7.07 *	9.10 *	7.99 *	1.57	7.7 *	0.71						12.9	1.34
10. Mutton	—	1.86	—	2.03 ¶	8.4	1.125‡	2.9	3.7	4.0	3.8	15.40	6.6	1.43
11. Pork	3.92	4.02	4.41	1.41	6.9	1.23‡	5.9	8.9	7.6	8.7	17.78	14.3	1.76
12. Veal	6.15	8.92	15.75	1.61	0.1	0.54	—	—	—	—	—	4.4	1.13
13. Sausage	4.53	3.82	2.96	1.77	5.5	1.40	—	—	—	—	—	3.3	3.09
14. Ham	1.01	0.98	1.24	3.23	—	—	—	—	—	—	—	—	—
15. Bacon	2.73	4.67	2.90	2.25	1.4	1.34	0.6	0.5	1.1	0.8	17.44	2.4	2.40
16. Potatoes	88.99	84.28	70.04	0.14	223.3	0.033	214.1	221.0	179.8	201.2	0.933	95.7	0.10
17. Sugar	31.78	35.01	36.76	0.47	31.1	0.30	10.8	12.4	13.5	13.0	7.52	30.9	0.46
18. Coffee	3.84	4.82	5.49	4.28	2.3	4.50	4.4	5.7	3.8	5.2	27.37	0.1	8.20
19. Tea	0.32	0.37	0.58	9.00	0.3	6.00	—	—	—	—	—	0.2	11.75
20. Cocoa	—	—	—	—	0.1	2.00	—	—	—	—	—	—	—
21. Cheese	4.77	6.08	6.78	1.77	0.1	2.50	0.5	0.8	3.0	1.0	22.96	1.3	1.91 +
22. Milk, unskimmed (litres)	106.60	115.53	116.30	0.29	154.9	0.14	309.6	379.9	395.1	378.0	1.67	172.3 §	0.22
23. Eggs (number)	132.41	158.69	192.23	0.11	7.6 §	0 065	25.0	43.4	111.6	58.5	0.914	126.0 §	0.13
24. Rice	—	1.76	—	0.75	4.3	0.56	—	—	—	4.2	6.02	2.0	0.65
25. Peas, dried	0.29	0.42	0.56	0.79	2.4	0.35	2.0	2.2	2.4	2.6	5.69	3.4	0.57
26. Beans, haricot	—	—	—	—	—	—						0.7	0.53
27. Fish, fresh	2.38	6.19	7.02	0.86 ¶	24.5	0.315‡	11.8	15.0	24.8	20.2	4.37 x	10.0 x	1.30
28. „ other	6.19	3.56	1.91	0.93 ¶	16.5	0.58 x	5.5	6.7	8.0	7.1	7.42 x	5.0 x	0.88
29. Prunes	—	0.96	—	1.38	—	—	—	—	—	1.9	14.09	—	—

Denmark: 4. Quantity of oatmeal.

Estonia: 4. Quantity and price of barley meal.
6, 10, 11, 27. Average price of two different qualities.
23. Eggs: 0.4 Kg. at 17.5 eggs a Kg.

Finland: 4. Quantity and price of oatmeal.
25,26. Price of peas only, for 1930.
16. Potatoes: The quantities are shown in litres in the enquiry; they have been calculated at the rate of 750 grammes the litre. Price = 0.70 Mcs. the litre.

Latvia: 4. Quantity of oatmeal and barley meal, price of barley meal.
9. 1st quality. Price of minced beef.
22. Milk: 177.6 Kg. at 1031 gr. the litre.
23. Eggs: 7.2 Kg. at 17.5 eggs a Kg.

DATA ON WHICH CALCULATIONS ARE BASED *(cont.)*

Articles	NORWAY				POLAND							
	Oslo				Warsaw						Upper Silesia	
	Annual consumption per unit, having an annual income of:			Average price in July 1930	Annual consumption per unit, having an annual income of:					Prices in Warsaw in July 1930	1927 Ann. consum. p. unit, ann. income of:	Prices in Katowice in July 1930
					1927			1928	1929			
	Kr. 1,170 workers	Kr. 1,430 workers	Kr. 2,450 workers		Zl. 500 workers	Zl. 800 workers	Zl. 1,500 workers	Zl. 900 workers	Zl. 900 workers		Zl. 900 workers	
	Kg.	Kg.	Kg.	Kr.	Kg.	Kg.	Kg.	Kg.	Kg.	Zl.	Kg.	Zl.
1. Bread, white	7.40	8.65	6.28	0.80	7.70	14.26	28.87	10.96	13.52	0.87	—	—
2. Bread, rye or " black "	106.49	95.13	75.50	0.37	147.46	153.64	130.02	156.02	145.11	0.43	29.78	0.38
3. Flour, wheat	15.44	14.84	20.85	0.36	17.99	22.41	26.17	21.58	21.31	0.92	11.86	0.80
4. ,, oat	—	2.45	—	0.48+	—	6.31	—	—	5.59	0.42	4.49	0.66
5. ,, rye	—	3.58	—	0.31	—	0.10	—	—	0.11	0.43	97.93	0.36
6. Butter, fresh or salted	0.93	3.54	9.53	3.42	1.41	2.55	5.05	1.79	1.61	4.88	1.05	6.60
7. Margarine	23.22	23.66	19.10	1.94	—	—	—	—	0.02	3.80	—	—
8. Lard	0.90	0.04	1.01	1.94	1.44	3.09	6.13	2.70	2.62	3.49	8.15	3.20
9. Beef, 1st quality	2.25	2.62	2.30	2.68	9.50	11.75	16.02	12.49	12.30	2.95	16.14	2.80
,, 2nd ,,	6.96	7.16	1.69	2.15								
10. Mutton	2.53	3.49	3.14	2.05	0.03	0.30	0.35	0.92	1.33	3.20	0.11	2.80
11. Pork	3.79	6.75	15.95	2.52	4.49	12.04	29.31	12.43	14.08	3.20	8.21	3.20
12. Veal	1.86	2.51	3.42	1.99+	0.04	2.83	3.89	1.69	1.24	3.04	0.24	2.80
13. Sausage	—	—	—	—	1.96	3.44	4.94	5.31	6.39	4.50	6.62	4.40
14. Ham	—	—	—	—	—	—	—	—	1.46	2.70	—	—
15. Bacon	6.37	6.79	5.81	2.21+	5.68	7.38	9.09	8.22	7.90	3.33	12.07	3.20
16. Potatoes	101.90	98.84	72.99	0.13	181.49	195.01	176.04	185.41	189.24	0.11	287.45	0.10
17. Sugar	26.67	25.35	39.09	0.62+	16.78	20.76	24.81	20.49	20.52	1.68	17.10	1.64
18. Coffee	5.80	5.92	5.55	3.61 ǂ	0.09	0.19	0.30	0.19	0.17	9.89	0.32	12.00
19. Tea	0.10	0.08	0.03	9.64	0.14	0.17	0.33	0.13	0.14	27.43	0.06	26.00
20. Cocoa	1.36	1.26	1.62	2.83	—	—	—	—	—	—	—	—
21. Cheese	8.66	9.54	14.82	2.05+	0.29	0.66	2.21	0.88	1.25	1.80	1.14	1.40
22. Milk, unskimmed (litres)	143.10	158.27	189.82	0.28	38.63	86.83	141.74	92.55	87.82	0.43	50.85	0.42
23. Eggs (number)	150.50	201.08	285.95	0.097	11.73 §	51.00 §	143.33 §	46.40 §	46.20 §	0.14	40.00 §	0.13
24. Rice	—	1.02	—	0.80	—	3.15	—	3.35	3.07	1.40	3.48	1.10
25. Peas, dried	2.70	2.21	0.82	0.58	1.78 *	1.86 *	1.45 *	1.81 *	1.80 *	0.43	2.68 *	0.56
26. Beans, haricot	0.33	0.31	0.00	0.64	1.58 *	1.65 *	1.29 *	2.13 *	1.98 *	1.50	2.53 *	1.00
27. Fish, fresh	14.97	16.82	20.97	0.78+	—	3.72	—	0.33	3.80	2.52	6.13	1.80
28. ,, other	6.14	6.39	3.48	0.84+								
29. Prunes	—	1.25	—	1.30	—	—	—	—	—	—	—	—

Norway: 4. Oat flour 0.58 Kg. at 0.57 Kr.
Oatmeal 1.85 Kg. at 0.45 Kr.
20. Quantity and price of chocolate.
23. Eggs: 11.49 Kg. at 17.5 eggs a Kg.
18. Average price of 2 different qualities.

Poland: Warsaw: 4. Quantities of meals, prices of barley meal.
23. Eggs: Kg. 0.67; 2.91; 8.19; 2.65; 2.64.

27-28. Quantities of herrings (9 to the Kg.) at 0.28 Zl. the herring.
7,-14. Prices used by " Contribution ", adapted for comparison by district basket.

Upper Silesia: 4. Quantities of meals, prices of barley meal.
23. Eggs: 2.28. Kg.
27,28. Herrings: 9 per Kg. at 0.20 Zl. the herring.

DATA ON WHICH CALCULATIONS ARE BASED *(cont.)*

Articles	SWEDEN									SWITZERLAND	
	Stockholm					Göteborg		Malmö		Winterthur	
	Annual consumption per normal household of 3.3 units, having an annual income of:				Prices in Stockholm in July 1930	Ann consum. p. nor. house, ann. income of:	Prices in Göteborg in July 1930	Ann. consum. p. unit, ann. income of:	Prices in Malmö in July 1930	Ann. consum. p. unit, ann. income of:	Prices in 34 communes in July 1930
	Kr. 3,700 p.u. 1,140	Kr. 5,200 p.u. 1,570	Kr. 7,500 p.u. 2,250	Kr. 1,600 p. unit		Kr. 4,100 p. u. 1,240		Kr. 1,217		Fr. 2,000	
	workers and lower-grade employees					workers and lower-grade employees		workers and lower-grade employees		workers and employees	
	Kg.	Kg.	Kg.	Kg.	Kr.	Kg.	Kr.	Kg.	Kr.	Kg.	Fr.
1. Bread, white	25.7	27.4	28.7	8.3	0.75	8.9	0.94	7.9	0.75	135.2	0.49
2. Bread, rye or " black "	101.6	101.6	105.3	30.8	0.64	76.9	0.60	91.9	0.76		
3. Flour, wheat	143.2	148.8	145.5	45.1	0.39	205.3	0.40	21.0	0.39	9.8	0.59
4. ,, oat	—	—	—	6.6	0.42	—	—	2.5	0.36	—	—
5. ,, rye	32.7	37.3	40.6	11.3	0.34÷	57.8	0.31+	1.3	0.30÷	—	—
6. Butter, fresh or salted	30.0	37.6	46.5	11.4	2.70	34.6	2.97	12.3	2.65	9.0	4.98‡
7. Margarine	19.8	19.8	21.5	6.0	1.39	21.4	1.39	7.1	1.39	2.4	2.05
8. Lard	6.9	6.9	5.9	2.1	2.00	4.0	1.60	1.8	1.50	4.4	2.84
9. Beef, 1st quality	4.5 *	5.3 *	6.4 *	1.6 *	2.12	33.0	1.70+	1.2	2.26	11.8	3.65‡
,, 2nd ,,	22.9 *	25.7 *	31.9 *	7.8 *	1.47			6.2	1.74		
10. Mutton	6.9	7.6	0.6	2.3	2.79	4.0	2.72	1.3	2.50	0.1	4.23
11. Pork	21.1	24.8	29.4	7.5	2.54	20.4	2.34	11.9	2.06	5.5	4.44
12. Veal	13.5	16.8	20.5	5.1	2.22+	10.6	1.62−	2.6	2.50+	2.9	4.68‡
13. Sausage	—	—	—	—	—	—	—	—	—	11.3	4.00 ¶
14. Ham	—	—	—	—	—	—	—	—	—	—	—
15. Bacon	8.9	13.2	16.8	4.0	2.56	12.9	2.38	3.7	2.20	3.1	5.32
16. Potatoes	301.0	365.3	432.3	110.7	0.13	373.6	0.12	114.6	0.10	87.6	0.17
17. Sugar	88.4	102.9	117.8	31.2	0.37	106.3	0.42	27.7	0.41	27.2	0.61
18. Coffee	17.5	19.8	24.1	6.0	3.52	19.8	3.40	6.4	3.63	3.9	3.48‡
19. Tea	0.26	0.33	0.53	0.1	8.00	0.5	9.00	0.1	9.00	0.1	8.76
20. Cocoa	1.7	2.0	2.3	0.6	2.25	2.0	2.00	0.7	2.25	—	—
21. Cheese	14.9	15.8	19.8	4.8	2.21	19.8	1.60	4.6	2.31	7.2	3.61
22. Milk, unskimmed (litres)	779.5	867.6	935.2	262.9	0.24	787.1	0.19	193.3	0.18	432.1	0.35
23. Eggs (number)	570.0	690.0	898.0	209.0	0.08	679.0	0.08	186.7	0.07	160.6	0.18
24. Rice	—	—	—	1.8	0.59	—	—	0.9	0.48	3.5	0.68
25. Peas, dried	4.5	5.3	4.6	1.6	0.30	4.1	0.34	1.6	0.40	4.2 *	1.01
26. Beans, haricot	1.7	2.0	1.7	0.6	0.46	1.5	0.45	0.8	0.51	3.5 *	1.01
27. Fish, fresh	37.6	47.2	58.7	14.3	0.88+	74.6	0.93	17.7	1.15	—	—
28. ,, other	26.4	33.0	38.6	10.0	0.76+	17.2	0.87	5.6	0.58	—	—
29. Prunes	—	—	—	1.6	1.30	—	—	—	—	—	—
30. Rusks	—	—	—	2.1	1.66	—	—	3.0	1.56	—	—
31. Milk, skimmed (litres)	—	—	—	2.5	0.09	—	—	5.2	0.08	—	—

Sweden: Stockholm: 4. Oat flour and meal.
Göteborg: Average price of 2 different qualities.
Malmö: 4. Oat flour and meal.

Switzerland: 7. Price of " edible fat ".
6, 9, 12, 2. Average price of 2 different qualities.

DATA ON WHICH CALCULATIONS ARE BASED *(concluded)*

	Czechoslovakia										
	Whole country 1927-1928				Prague	Prague 1927-1928		Prague 1928-1929		Prague 1929-1930	
Articles	Annual consumption per unit having an annual income of:				Prices in Prague in July 1930	Ann. consum. p. unit, ann. income of:	Prices paid in 1927-1928	Ann. consum. p. unit, ann. income of:	Prices paid in 1928-1929	Ann. consum. p. unit, ann. income of:	Prices paid in 1929-1930
	Kc. 5,800	Kc. 6,540	Kc12,300	Kc20,400		Kc. 7,140		Kc. 7,600		Kc. 8,100	
	workers		officials			workers		workers		workers	
	Kg.	Kg.	Kg.	Kg.	Kc.	Kg.	Kc.	Kg.	Kc.	Kg.	Kc.
1. Bread, white	33.06 *	40.93 *	28.51 *	24.61 *	2.43	82.16	3.56	85.91	3.45	87.72	2.91
2. Bread, rye or " black "	53.58 *	66.32 *	46.21 *	39.87 *	2.24						
3. Flour, wheat	54.87	59.22	50.81	50.41	3.00 ‡	59.39	4.34	58.10	4.05	55.40	3.54
4. ,, oat	—	—	—	—	—	—	—	—	—	—	—
5. ,, rye	—	—	—	—	—	—	—	—	—	—	—
6. Butter, fresh or salted	5.84	4.44	8.32	9.86	25.00	5.29	30.69	4.96	30.53	5.27	29.06
7. Margarine	3.00	3.31	2.04	2.88	14.00	1.12	16.09	1.33	16.41	2.32	15.90
8. Lard	11.82	12.48	10.51	9.92	14.00	14.71	15.93	13.37	15.97	12.68	16.57
9. Beef, 1st quality	3.38 *	3.67 *	4.24 *	4.58 *	16.00	15.93	15.28	15.26	14.92	15.69	15.54
	T 12.41	T 13.46	T 15.55	T 16.79	13.09+						
,, 2nd ,,	9.03 *	9.79 *	11.31 *	12.21 *	12.00						
10. Mutton	0.92	0.80	1.26	1.85	16.00	1.45	13.19	1.00	14.20	0.95	13.49
11. Pork	9.40	9.58	12.41	9.62	18.00	13.76	17.19	14.52	16.72	15.14	18.30
12. Veal	1.35	1.84	5.24	8.00	14.00	2.72	15.34	3.73	15.64	3.90	17.81
13. Sausage	1.24	2.64	5.00	6.49	21.00 ‡	4.13	20.37	3.93	20.58	4.65	21.79
14. Ham	—	—	—	—	—	—	—	—	—	—	—
15. Bacon	0.82	1.25	0.81	0.70	23.00	0.32	20.22	0.35	21.54	0.42	21.00
16. Potatoes	102.62	105.48	77.52	66.39	1.20	82.04	1.07	76.39	0.952	84.74	0.796
17. Sugar	32.02	31.73	30.50	26.58	6.40	31.28	6.00	30.40	6.20	30.82	6.35
18. Coffee	1.09	1.06	1.43	1.48	46.00	1.36	51.18	1.37	50.83	1.46	50.13
19. Tea	—	—	—	—	—	—	—	—	—	—	—
20. Cocoa	—	—	—	—	—	—	—	—	—	—	—
21. Cheese	—	5.68	9.57	—	6.00	—	—	—	—	—	—
22. Milk, unskimmed (litres)	221.61	182.39	217.32	240.42	2.20	195.33	2.155	185.00	21.8	196.55	2.22
23. Eggs (number)	151.52	180.56	276.46	314.63	0.70	235.00	0.804	214.26	0.848	222.93	0.898
24. Rice	—	5.50	5.17	—	3.90+	5.57	4.58	4.97	4.49	4.68	4.33
25. Peas, dried	1.81	2.76	2.73	3.40	4.90 ‡	3.59	5.64	2.73	8.07	2.46	6.20
26. Beans, haricot											
27. Fish, fresh	—	—	—	—	—	—	—	—	—	—	—
28. ,, other	—	—	—	—	—	—	—	—	—	—	—
29. Prunes	—	—	—	—	—	—	—	—	—	—	—
30. Rusks	—	—	—	—	—	—	—	—	—	—	—
31. Milk, skimmed (litres)	—	—	—	—	—	—	—	—	—	—	—
32. Game	—	—	—	—	—	4.38	15.82	5.34	16.52	5.05	16.31
33. Meat, other *(Selchwaren)*	—	—	—	—	—	8.01	18.20	8.66	18.07	9.85	18.91
34. Fats, other *(Kernfett)*	—	—	—	—	—	1.13	11.26	1.39	11.49	1.07	10.93
35. Confectionery *(Weissgeback)*	—	—	—	—	—	17.45	7.40	19.21	7.01	21.13	6.58
36. Meal	—	—	—	—	—	3.88	4.73	3.59	4.61	3.46	4.19
37. Chicory	—	—	—	—	—	4.75	4.74	4.29	4.29	4.10	4.85

Czechoslovakia:

Whole country: 3. Average price of 2 different qualities.
13. ,, ,, of 4 ,, ,,
21. Cheese: Quantities obtained by dividing the expenditure mentioned at the enquiry by average price published for the period of enquiry 1927-1928.

Prague: Prices for the years 1927-1928, 1928-1929, 1929-1930 obtained by dividing the expenditure for each article by the quantity given in each case.

APPENDIX IV

COMPOSITION OF REGIONAL BUDGETS

Annual consumption per consumption unit in kilogrammes, except for milk and eggs

Articles	First system of regions					Second system of regions			
	Regions including					Regions including			
	Stockholm, Helsinki, Oslo	Copenhagen, Berlin, Warsaw, Brussels	Copenhagen, Berlin, Warsaw	Warsaw, Riga Tallinn	Prague, Vienna, Switzerland	Oslo, Stockholm, Copenhagen, Helsinki	Berlin, Vienna, Switzerland, Brussels	Berlin, Vienna, Switzerland	Prague, Warsaw
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bread, white	17.5	18.07 [1]	18.07	36.27	104.43 [10]	18.24	131.75 [10]	110.53 [10]	123.18 [10]
Bread, rye or " black "	59.0	105.13 [1]	105.13	138.24	—	65.35			
Flour, wheat	31.2	13.59 [2]	17.23	14.37	29.28	27.33	12.45 [2]	15.72	38.36
,, rye	15.8	—	—	—	—	—	—	—	—
Oat or barley (flour, meal or flakes)	7.3	—	—	6.96	—	6.48	—	—	—
Butter, fresh or salt	11.2	8.54	5.60	5.87	6.48	10.76	9.35	6.69	3.44
Margarine	10.7	7.50 [4]	8.47	—	2.13	11.94	4.50	4.59	1.17 [4]
Lard	—	3.85	3.43	2.21	6.46	1.85	4.08	3.73	7.65
Beef	12.6	11.88	10.01	13.30	15.90	12.30	13.98	12.81	14.00
Mutton	3.2	0.98 [2,5]	1.26	5.44	0.41	2.86 [5]	0.25	0.29	1.14
Pork	7.7	8.43	8.63	11.76	13.22	6.74	10.03 [2]	10.77	14.61
Veal	—	3.55	3.89	1.91	3.13	—	2.38	2.33	2.57
Sausage	—	6.95 [3]	7.87	5.06	—	—	10.05 [3,9]	12.00	5.52
Bacon	3.9	5.68	5.19	3.90	2.79	4.07	4.53	3.65	4.16
Ham	—	1.11 [3,4]	1.05	—	—	—	—	—	—
Potatoes	136.9	159.51	142.37	169.41	71.97	123.76	123.92	94.92	136.99
Sugar	23.2	21.65	24.04	27.51	27.24	26.14	20.49	22.50	25.67
Coffee	5.7	3.16	2.06	0.86	3.48	5.49	4.16	3.39	0.82
Tea	—	0.18 [2]	0.20	0.21	—	—	0.11 [2]	0.11	—
Cheese	5.1	4.28	4.04	0.88	5.34 [6]	5.36	5.04	5.05	3.47 [6]
Milk (litres)	266.4	122.00	117.82	138.34	265.21	228.68	220.93	249.73	142.19
Eggs (number)	156.2	136.25	119.07	59.93	207.84	156.90	185.10	184.20	134.57
Rice	2.3	2.89 [2]	2.24	3.12	4.13	2.20	3.61 [2]	3.20	3.88
Peas and Beans	2.4	2.48 [7]	2.30	2.53 [8]	3.83	1.94 [7]	3.69	3.91	3.12
Prunes	1.6	—	—	—	—	1.43	—	—	—
Fish, fresh	17.1	—	—	—	—	14.38	—	—	—
,, other	7.8	—	—	—	—	6.76	—	—	—
Herrings	—	—	—	6.70	—	—	—	—	—
Number of articles	22	22	22	22	18	22	20	20	19

Notes to Table:

[1] "Black" bread being not generally consumed in Belgium, the quantities of white and "black" bread given in column (2) do not include Brussels. For Belgium, the quantity of 195.41 kilogs. of white bread has been taken as representing the total consumption of this commodity.

[2] For Belgium, the quantities of flour, mutton, tea and rice included in the averages have been taken from the "Contribution". The figures in question refer only to Antwerp.

[3] For Belgium, the prices taken for these quantities are those given in the "Contribution" (report for Antwerp) in October 1930. In order to make allowance for the reduction in the prices of the commodities between that date and January 1931 (the period to which all the other prices for Brussels refer), these figures have been multiplied by the coefficient 0.935.

[4] The prices given for Warsaw are taken from the "Contribution". They refer to July 1930, as do all other prices given for Warsaw.

[5] For Copenhagen, the price of mutton given for this quantity is an estimate obtained by applying the ratio of the price of mutton to the mean of the prices of beef, veal and pork (as shown in the budgetary enquiry) to the same mean calculated for July 1930.

[6] The quantity of cheese for Prague is taken from the budgetary enquiry of 1927-1928, the other quantities for Prague referring to 1929-1930.

[7] The quantity considered for Copenhagen represents the consumption of peas only.

[8] This quantity represents only dried peas for the 3 towns included in the average.

[9] The price of sausage meat in Switzerland has been calculated in the same way as that adopted for mutton in Copenhagen (cf. note 5 above).

[10] With the exception of Brussels (where no "black" bread is eaten) and Switzerland (where only one quality of bread is given, the price referring to grey bread), the prices given for these quantities are weighted averages (based on the quantities consumed *in each town*) of the prices of white bread and "black" bread.

General note:

The quantities shown above are the simple averages of the quantities consumed per consumption unit (adult male) having an average income, in the towns included in the region. The composition of the regions is that given on page 54 of the study. It is to be noted, however, that the figures given in column (4) have not been repeated in the 2nd system. Again, in addition to columns (2) and (7) where the Belgian averages are included in the average, columns (3) and (8) where Belgian quantities are excluded, have also been inserted. This question is dealt with on pages 63-64 of the report. It may be added that the utilisation of the budget of column (8) instead of that given in column (7) gives the following results (cf. table XIX):

Localities compared with Berlin	Index numbers of the Berlin region (2nd system) according to the quantities of the columns	
	(7)	(8)
Vienna	102.26	101.39
Switzerland	106.98	107.29

Here again, the exclusion of Belgium has only a negligible influence.

It should be noted that the prices given for Brussels refer to January 1931, whereas those for all other localities date back to July 1930. The quantities used in the calculation of the averages for Prague and Warsaw are those for 1929-1930 and 1929 respectively, although those used for all other calculations for these two towns refer to 1927-1928 and 1927 (cf. page 63 above).

PART II

INTERNATIONAL COMPARISON OF RENTS

By R. GUYE

I

INTRODUCTION

The study of the problem of the international comparison of rents as a factor in cost of living has hitherto been comparatively summary. When the International Labour Office first set out to enquire into the purchasing power of real wages in different towns, it made a " correction " in its indexes of real wages in order to allow for the expenditure on rent. But this correction was not based on the absolute amount of rents, since data could not be collected then, but on the difference between the relative level of rents at the date considered as compared with the pre-war level and the corresponding relative level of retail food prices as shown by the national cost-of-living index numbers [1]. The object of the correction was to take into account the fact that at that time rents, which were subject to restrictions in most countries, had fluctuated in a manner that differed widely from the fluctuations of food prices. It suffered, however, from an element of arbitrariness in that it was based on the hypothesis that before the war the relation between rents and food prices was the same in different towns or different countries. With the gradual abolition or relaxation of rent restrictions in several countries, and the reduction of the discrepancy between the levels of rent and of prices, this correction no longer appeared justified, and was therefore given up in 1929.

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[1] Cf. *International Labour Review*, Vol. X, No. 4, Oct. 1924, pp. 646-647.

In another study undertaken more recently by the International Labour Office [1], comparisons were attempted on the basis of actual rent figures. These comparisons were, however, of a somewhat special nature, due to the particular object of the enquiry, which was to ascertain the expenditure needed to obtain in thirteen European towns a standard of living approximately equivalent to that of the workers of the Ford Motor Company in Detroit. The prevailing type of dwelling for working-class families in Detroit (a house of 4-5 rooms with bathroom) was however comparatively rare in many European towns, or quite definitely belonged to the class of house not occupied by workers' families. A twofold comparison was therefore made, by relating rents in Detroit on the one hand with dwellings which corresponded in type and quality as closely as possible to those in Detroit, and on the other with dwellings of the same size (4-5 rooms) but of the predominant quality in each town. The method did not prove completely satisfactory, however, for in certain towns where the standard of housing of the workers was particularly low, the dwellings of the size mentioned were to be found chiefly among those occupied by middle-class families, and thus came under the heading of a rent category that was, comparatively speaking, too high [2].

The few other enquiries that have been made by various public or private institutions are based as a rule only on somewhat limited material, which seriously affects the significance of the results [3]. The only exceptions are the enquiries undertaken from 1905 to 1909 by the British Board of Trade [4] for comparing the cost of living of families in Great Britain, Germany, France, Belgium, and the United States. Rent figures, based chiefly on estimates, were collected for dwellings of three or four sizes (varying from two to six rooms according to country); a measure of the general level in each country was obtained by taking the simple arithmetic average of the median rents for dwellings of the different sizes, and this was compared with an average calculated for Great Britain for dwellings of the sizes corresponding to those considered in each country. In certain countries where these data did not appear very satisfactory, supplementary estimates were made by using the available material for rents collected for the general cost of living index numbers. But however interesting this information may be [1], its date is clearly too distant to satisfy present needs.

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[1] *A Contribution to the Study of International Comparisons of Costs of Living*, Studies and Reports, Series N, No. 17; see in particular pp. 17-19.

[2] In this connection see *ibid.*, pp. 243-246, " Note on the Relation between the Index Numbers of the Cost of Housing and the Cost of Food in the Different Cities ", by R. GUYE, or the *International Labour Review*, Vol. XXVI, No. 3, Sept. 1932, pp. 360-363.

[3] Mention may be made of that undertaken in January 1930 by the Unilever Company (" International Middle-Class Living Costs ", *The Economist*, Nov. 8, 1930, pp. 846-848), in which the data of expenditure on housing, comprising rent, rates and taxes connected therewith, insurance premiums, water rates and the cost of heating and lighting, were collected for one or more towns in eight European countries on the basis of information supplied by employees of the Company earning an " equivalent " salary.

[4] " Reports on an Enquiry by the Board of Trade into Working-Class Rents, Housing and Retail Prices, together with the Rates of Wages in Certain Occupations in the Principal Industrial Towns of the German Empire, France, Belgium and the United States ", Cd. 3864, 4032, 4512, 5065, 5609. Reports

This being the state of the question, the first thing to be done was therefore to collect adequate statistical material on rents in different countries. This was attempted by the International Labour Office during its recent annual enquiries into the retail prices of certain foodstuffs and articles of heating and lighting in some 80 towns, carried out in connection with its enquiry into wages in different occupations. The work was not without difficulty, and it was not until the enquiry of October 1932 that the results obtained could be considered sufficiently satisfactory for publication [2].

The data so collected were, however, far from being completely satisfactory; but it has still been thought worth while to consider even now how these data—or slightly improved data—might be used for constructing international index numbers of rents that would ultimately supplement the international index numbers of food costs at present being published or studied by the Interna-

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summarised in the *Board of Trade Labour Gazette*, Feb. and July 1908, March 1909, March 1910 and April 1911.

[1] Similar data were collected in the enquiry undertaken in 1914 by the Economic Commission of the Government of the Union of South Africa, and together with the statistics available for other British Dominions have served as a basis for estimating the average rent per living room in these different countries. (*Union of South Africa: Report of the Economic Commission*, Jan. 1914.)

[2] Enquiry of October 1932 (with an account of the principles governing the investigation). *International Labour Review*, Vol. XXVIII, No. 2, Aug. 1933: " International Statistics of Rents in Certain Towns in 1932 ", pp. 248-264.

Since the time of writing a new enquiry has been undertaken, in October 1933, on the same lines as that of 1932. The results, which show marked progress, were published in the *International Labour Review*, Vol. XXIX, No. 6, June 1934: " Retail Prices and Rents in Certain Towns and Countries in October 1933 ", pp. 859-865.

tional Labour Office[1]. Hence the present enquiry, which of course is purely preliminary in character.

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[1] This would not, moreover, be the only value of good international statistics of rents; it would be of great value for a study on housing policy in general. When the Office made an enquiry on this subject, although a large amount of statistical information was assembled on the principal aspects of the question, the problem of rents, though a fundamental one, could only be touched upon owing to the lack of exact data. See INTERNATIONAL LABOUR OFFICE: *Housing Policy in Europe*, Studies and Reports, Series G, No. 3, Geneva, 1930; also *International Labour Review*, Aug.-Sept. 1931: " Post-War Difficulties and Housing Policy in Europe ", by Robert GUYE.

II

THEORETICAL EXAMINATION

There can be no question here of examining the whole of the problem of the construction of index numbers, or of discussing which of all the possible formulae for compiling index numbers would be most appropriate for the international comparison of rents as a factor in international comparisons of the costs of living of working-class families. In view of the fact that the rent index numbers, for which it is proposed here to find a method of compilation, are intended to supplement the Office index numbers of food costs, the methods used or studied by the Office for constructing those index numbers will be taken as a starting point.

These methods, it will be remembered, are:

(*a*) the method of standard weights, sometimes called the " basket of provisions " (used hitherto in the periodical publications of the Office);

(*b*) the method based on Fisher's so-called " ideal " formula (taken as a basis for the recent research of the Office, whether in the study mentioned above[1], or in Part I of this volume);

(*c*) a mixed method (contemplated in Part I of this volume) combining the two preceding methods and making use of the method of standard weights within regions of so-called " good comparability " (standard changing with each region) and of the " ideal " formula as a means of linking up the towns taken as the base or " centre " of each region, thus forming a continuous series of index numbers. Whichever of these methods is adopted, the problem of the international comparison of housing costs may be attacked from two quite different standpoints.

One of these consists in considering expenditure on housing in the same light as expenditure on any other article of subsistence

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[1] *Contribution to the Study of International Comparisons of Costs of Living.*

(bread, potatoes), and in treating it in exactly the same way. Thus in the " budget " or " budgets " used for weighting the index numbers, the " quantity " and " quality " of housing " consumed " would be determined just like the quantity and quality of bread and potatoes. The operation would therefore be that of recording in the different towns the price or rent of dwellings of the size and degree of comfort decided on, and to include this rent directly in the budget (while making the necessary adjustments for reducing the rent per family to the basis of " the unit of consumption " and for the fact that the expenditure on the items of food included in the index number forms only part of the actual expenditure of a working-class family on food, whereas rent represents the total expenditure on housing).

This system meets with two fundamental obstacles. In the first place, there is a practically complete lack of data for determining, on bases corresponding to those used for weighting items of food, the type or types of dwelling to be considered. For the family budget enquiries that serve as a basis for determining the weights for indexes of food costs seldom give particulars as to the quantity and quality—size and comfort—of the dwellings occupied by families whose food consumption is being analysed. As a rule only the expenditure on housing is recorded, but usually without indicating the kind of dwelling in question [1].

In the second place, it would be necessary to make quite sure that the type of dwelling selected for each town was in fact that needed for the comparisons, for the statistics collected for the Office enquiry into rents show that the relation between rents in different towns varies considerably with the type of dwelling[2]. The indexes would be without significance except for families occupying the selected types of dwelling, and this would both limit the interest of the enquiry and render its results uncertain.

In view of this twofold difficulty, the problem has been attacked from another standpoint, which it is believed will enable positive results to be obtained more rapidly and with greater certainty.

As expenditure on the " item " housing in family budgets is greater than that on any other single item, it is considered that, in order to determine the relation between the costs of this item in different towns, recourse should be had to more refined methods than in the case of items that play a comparatively insignificant part in the family budget. Just as when determining the " average price " of certain more important foodstuffs (bread, meat, potatoes), it was found necessary to take into account the prices charged in different districts, during different seasons, or for different qualities, so in a comparison of rents it is necessary first to establish the " average " rents in each town, account being taken of the rents of different types of dwellings and the special housing conditions in each locality.

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[1] It would no doubt be possible to have recourse to other sources, for instance housing censuses. But in this way it would hardly be possible to collect information relating to the same social classes and the same standards of income as those considered for the budget enquiries in regard to food; and this would make the results singularly unreliable.

[2] See tables IV and V.

Once this first stage has been properly settled, it will be possible to consider how to combine the indexes of housing costs with those of food costs. For this second stage the family budget enquiries are able to supply useful information, since for the combination of the two group indexes (food and housing) it will be sufficient to know the proportion of expenditure devoted to each group in the family budgets. Most of the enquiries on which the weights for the food cost indexes are based are in a position to supply this kind of information, sometimes with a great deal of detail, distinguishing the size of the family and the level of income.

The present study is confined to this first stage of the operation, the second being reserved for subsequent research as it must be considered in relation to the solutions adopted for calculating the index numbers of food costs.

Conceived on these lines, the international index numbers of rents will serve to measure the relation between housing costs in different towns. This average housing cost—to take up the parallel with food costs—will be the average cost of a specified quantity of " housing accommodation ", account being taken of the different kinds and different qualities ordinarily " consumed " in the various towns, and the amount consumed in each case. In other terms, it will be the average of the rents paid for a given " unit of housing ", account being taken of dwellings of different sizes and different degrees of comfort, and their relative importance in the prevailing housing conditions.

When the compilation of international rent index numbers is considered in this light, three fundamental points have to be examined:

(*a*) the determination of comparable standard units of housing for which the rent data have to be collected in order to arrive at an average;

(*b*) the basis for determining, when calculating the averages, the importance to be attached to each standard unit of housing, that is to say, the basis for the weights;

(*c*) the method of weighting itself.

On the first two points, decisions were already reached when organising the enquiry into rents in 1932, and the article giving the results of that enquiry indicated the ground for these decisions. The matter will therefore be dealt with only briefly here, but it may be useful to check the validity of the position adopted in the light of the figures obtained. The third point, on the other hand, calls for more detailed examination, which will be supplemented by certain experimental calculations based on the material obtained from the 1932 enquiry.

Determination of Standard Units of Housing

The unit of housing adopted for the rent statistics that were based on the results of the 1932 enquiry was the " dwelling ". This was, in fact, the natural unit for the collection and compilation of the data since the rents themselves were fixed on this basis.

For the construction of an index number which is to rest on the average of the rents of dwellings of different kinds, it is necessary to adopt a unit which represents a quantity of housing accommodation that will be as uniform as possible. To take the dwelling as a unit is clearly unacceptable, since its size, that is to say the quantity of housing accommodation, may vary considerably. To take the average of the rents of different dwellings would be much the same thing as to calculate the average price of loaves of bread without considering the weight of the loaves.

The unit of housing to be adopted should be, therefore, the room. It is true that from the quantitative point of view it is not as strictly defined as the units of weight and volume (e.g., the pound, the gallon) used for indexes of food costs, but housing statistics are still far from adequate for determining the amount of rents on bases that would correspond to these units of quantity (e.g. the cubic air space, the habitable area of the dwelling).

It is also true that the room is a very theoretical unit, for, in fact, it cannot be separated from the dwelling to which it belongs. Its character would be substantially altered by an attempt to isolate it, for since it belongs to the dwelling, it is in fact connected with other rooms and the domestic equipment (kitchen, bathroom, offices, heating and laundry equipment, etc.) without which its utility value would be considerably reduced. It is therefore proposed, not to determine the " theoretical " rent of a room taken by itself, but merely to relate the rent of dwellings of different kinds or qualities to a sort of common denominator, a relatively fixed quantitative unit of housing accommodation, namely, the room. The rent per room being obtained by dividing the rent of the dwelling by the number of rooms, each room is thus assigned an average share of the price representing the advantages derived from the fact of belonging to the dwelling.

For the 1932 enquiry dwellings were classified into 12 categories based on two fundamental criteria: size, measured in fact by the number of rooms; and " comfort ", measured by the presence or absence of a bathroom[1].

This distinction according to size thus serves two purposes. On the one hand, it makes it possible to reduce the rents to a common denominator, namely, the room. On the other, it makes it possible to distinguish between the rents per room according to a fundamental quality of the dwelling to which the room belongs, namely, the size of the dwelling. For, given a particular family, it is a question of greater or less comfort or convenience to occupy a larger or smaller dwelling[2].

A brief examination of the results of the enquiry—certain parts of which are summarised in tables IV and V below—clearly shows that this twofold distinction according to the size and comfort of dwellings is essential.

Given a certain degree of comfort, the difference between the rents per room for the different sizes of dwellings is everywhere substantial; in certain cases it is as much as 50 per cent. between the two extreme categories. It may be noted that in many cases these variations, which form a curve reaching its minimum for the rents of average-sized dwellings, display a fairly characteristic tendency. The relatively higher level of the rents of quite small dwellings—one or two rooms according to circumstances—is no doubt to be explained by the fact that the cost of installing kitchen, lavatory, running water, heating, etc.,—in some cases a bathroom—is spread over the rent for the whole of the dwelling and is therefore all the heavier per room when the number of rooms is small. The difference is particularly marked in regard to one-roomed dwellings. The tendency is, however, counterbalanced for dwellings of a certain size—four, five or six rooms according to circumstances—by the fact that dwellings in these categories tend to belong also to the categories of increasing comfort and amenities [1].

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[1] In fact, the enquiry provided for only 10 categories, rent data being asked only for 5 sizes (dwellings of 1 to 5 rooms). Several countries, however, supplied figures for dwellings of 6 rooms, and experience shows that, to judge from the conditions prevailing in certain countries, future enquiries ought to be extended to comprise this category too.

[2] The size is therefore regarded here only from the point of view of quality, Its quantitative aspect is not taken into account until the later stage of the operations, when it becomes necessary to determine how much housing accommodation—that is to say, how many rooms—must be taken in the final budget for the general cost-of-living index number. It may be added that for this stage the possibility has been considered of making use of a basis of a different kind: the proportion of expenditure on housing in the family budget.

Rents also differ widely in dwellings which, though otherwise equal in size, are with or without a bathroom. Data in this connection are rarer, but for the few towns for which they are available [2] the point is very clear. The differences are too large to be due to the mere fact of the presence or absence of a bathroom. They show rather that the presence of a bathroom must be interpreted as showing that the dwelling by its general nature and equipment as a whole belongs to a higher general category than the rest.

No doubt it may be objected that the criteria of size and comfort adopted for these statistics are very rough and leave out of account many divergences which are nevertheless of importance from the point of view of the quality of the dwelling and therefore of its rent. This point was considered when the results of the 1932 enquiry were published. While its interest was recognised, it seemed that, before wishing to make still further refinements to meet the requirements of international rent statistics, it would be more useful to try first to obtain data satisfying fully the more general requirements laid down at the outset. The many gaps still to be found in the tables show that there is much room for progress in this direction.

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[1] Another fact to be noted is that it is not always the most usual category of size that shows the lowest rent per room. It is true that the available data concerning rents (table IV) and the distribution of dwellings by size (table IX) are not sufficiently comparable to enable definite conclusions to be drawn (the rent statistics relate to certain categories of dwellings—workers' dwellings or all dwellings; dwellings with or without bathroom—while the statistics of distribution by size relate to all dwellings—existing or occupied—in the town in question irrespective of the presence or absence of a bathroom).

[2] Cf. in the complete results of 1932 (*International Labour Review*, Aug. 1933) the data for Madrid, Utrecht, Zurich and the five British towns. Cf. also the results of the 1933 enquiry (*ibid.*, June 1934) carried out since the completion of this study and giving data of this kind for a larger number of towns.

When improvements have been obtained in this respect it will be time to think of perfecting the statistics by introducing greater precision in regard to the size of rooms and the comfort, hygiene and amenities of the dwellings. The experience gained from certain local enquiries suggests the direction that might then be taken and the manner in which these different factors might be taken into account [1].

In the international rent statistics published in the *Review* a third distinction was made, namely, between the figures for workers' dwellings properly speaking and the more general figures covering all dwellings. This is a distinction of quite a different kind. It does not result from a difference in the nature of the dwellings, but is rendered necessary by a difference in the scope of the available data. In principle only the data limited to workers' dwellings correspond strictly to the purpose of the enquiry, which is to provide the data for international comparisons of the costs of living of workers' families. It must be remembered that the general data relating to all dwellings have been collected only in the absence of data on workers' dwellings. Clearly these two types of data are not to be confused [2], for the difference between them is marked and varies from one case to another, as will appear from the few examples given in tables I (A) and I (B) below, which have been drawn from the results of the 1932 enquiry and other statistics.

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[1] Reference may be made in particular to two enquiries, one undertaken in Sweden in 1930 by Mr. Alfred PETTERSON and Mr. A. STEENHOFF of the Hygiene Section of the Stockholm School of Medicine, into rural housing conditions in Sweden (KAROLINSKA INSTITUTETS HYGIENISKA AVDELING: *Bostadsförhallandena pa landsbygden i Sverige*, Stockholm, 1930), the other in Germany in January 1909 by Dr. Friedrich RITZMANN, Chief Factory Inspector in the Grand Duchy of Baden, into the housing conditions of workers in the Gritzner engineering works at Durlach (*Einkommens- und Wohnverhältnisse der Arbeiter der Maschinenfabrik Gritzner A.-G. in Durlach*, Beilage zum Jahresbericht des Gewerbeaufsichtsamts für das Jahr 1913, Karlsruhe, 1914). In both these enquiries schedules were established for assigning a certain number of " points " to each dwelling according as it possessed or lacked certain characteristics. By this procedure it was possible not only to pick out dwellings which corresponded on all these different points, but also to establish cases of equivalence of the quality of dwellings on the assumption that the presence of certain of the specified advantages might compensate for the absence of others.

[2] It is true that this distinction is not of the same importance everywhere. In certain towns, especially in Eastern Europe, the differentiation between workers' dwellings and middle class or luxury dwellings can be made with sufficient certainty precisely on the basis of the size of the dwelling; thus, even where the rent statistics relate to all dwellings, we may fairly reasonably assume that the rents of the small dwellings are those of workers' dwellings and the rents of the medium-sized or large dwellings those of middle class or luxury dwellings. This cannot be accepted as a general rule, however, for in many countries the size of the dwelling is by no means a criterion of the

TABLE I (A). — MONTHLY RENTS OF WORKERS' DWELLINGS AND ALL DWELLINGS IN CERTAIN TOWNS

Size of dwellings (No. of rooms)	Copenhagen (1930)		Prague (1929)		Oslo (1931)	
	Workers' dwellings	All dwellings	Workers' dwellings	All dwellings	Workers' dwellings	All dwellings
	Kr.	Kr.	Kč.	Kč.	Kr.	Kr.
1	20.92	22.00	65.83	88.67	—	—
2	37.50	38.25	113.58	177.25	23.83	25.92
3	51.17	54.42	210.58	280.08	38.92	43.00
4	59.67	67.00	—	—	55.50	64.25
5	104.17	131.75	—	—	72.50	96.00

SOURCES. — Prague: CENTRAL STATISTICAL OFFICE, Dr. Jan KONOPAČ: *Les loyers à Prague de 1923 à 1930, comme élément du coût de la vie d'une famille ouvrière et d'employés*, p. 35 (data relating to dwellings of the municipality of Prague). — Copenhagen: STATISTICAL DEPARTMENT: *Loyers et logements*, Nov. 1930, p. 130. — Oslo: Figures taken from table IV.

The above remarks show clearly that the various distinctions made in the enquiry of October 1932 as to the type of dwellings and nature of the data do in fact correspond to differences in rent that are often substantial. It may, no doubt, be noted that to judge from the available figures, certain types of dwelling (according to size or comfort) represent an almost negligible proportion of the existing dwellings in certain towns [1]. If it were only a question of establishing the average level of rents in a town, it

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social class occupying it, either because the working class is reaching a high enough standard of living to occupy relatively large dwellings or, on the contrary, because the middle class has got into the habit of occupying relatively small dwellings which, however, have all modern conveniences. This latter tendency appears quite clearly in certain countries owing to the rise in building costs or the desire to allocate a larger share of family expenditure to other items. In this case the distinction between workers' dwellings and all dwellings is indispensable.

[1] Figures giving the distribution of dwellings by size, cf. table IX below; figures giving the distribution of dwellings according to the presence or absence of a bathroom, cf. in the afore-mentioned article of the *Review* (analysing the results of the 1932 rent enquiry) the notes appended to the table.

TABLE I (B). — MONTHLY RENT PER ROOM OF DWELLINGS CORRESPONDING TO DIFFERENT SOCIAL CLASSES IN CERTAIN ITALIAN TOWNS (SECOND HALF OF 1932) [1]

Towns	Dwellings situated within the town (but not in the centre)			Dwellings situated on the outskirts of the town (but not in suburbs)		
	Higher class	Medium class	Cheap	Higher class	Medium class	Cheap
	L.	L.	L.	L.	L.	L.
Florence	60	60	20	55	40	18
Genoa	60	58	36	—	45	31
Milan	97	80	57	77	72	52
Rome	100	100	85	90	80	65
Trieste	70	60	36	65	50	32
Turin	55	55	50	38	38	38

[1] The different types of dwellings are defined according both to their general characteristics (comfort, amenities, etc.) with reference to the customs of each locality, according to their size (viz.: higher class, over six (living) rooms; medium class, four to five rooms; cheap class, one to three rooms).

SOURCE. — FEDERAZIONE NAZIONALE FASCISTA DELLA PROPRIETA EDILIZIA: *Il mercato edilizio* (first quarter 1933, pp. 124-135).

would be unnecessary to collect data for these types of dwelling. But it is to be expected that these unusual types in one town will just be those that predominate in other towns, and that if it is desired to make comparisons between equivalent types in different towns, it will be necessary to collect data even for the more unusual types in any particular town.

BASIS OF WEIGHTING

When calculating index numbers the importance to be assigned to the different types of dwellings for which rents are recorded should normally rest on what may be called " consumption habits ", just as (to return to the parallel with indexes of food costs), account is taken, when calculating the relative prices of bread or meat, of consumption habits for determining the importance to be assigned to different qualities or kinds of bread or meat [1].

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[1] The idea of " consumption habits " is however not exactly the same for food and for housing. In regard to housing, each family occupies, that is to say " consumes ", only one dwelling and the consumption habits are indicated by the distribution of the different types of dwelling over the different families

The only data at present available to express the " consumption habits " in regard to housing are those drawn from housing censuses concerning the proportion of existing dwellings [1] belonging to the different types of size and comfort considered. They give an idea of the " consumption " at a particular time.

Since, however, these figures express the proportion of dwellings of different types, and the unit of housing adopted for determining the rent is the room, it may be asked whether it would not be appropriate to convert them all into terms of the same unit, the room.

But, if an average of the rents per room were taken, weighting them by the number or proportion of rooms, the result would be the average rent per room for all the rooms existing in the town or occupied by the working class. This would automatically give much greater importance to the rents of rooms in large dwellings [2], and the average would not have the desired significance. It should be remembered that the result ought to express the average cost of " a unit of housing accommodation ", account being taken of the different dwellings " consumed " by the families in the town or of the working class. To do this, the rents per room must be weighted by the number or proportion of dwellings of different types to which the rooms belong (the question of the quantity of housing accommodation made use of by each family being reserved for a subsequent stage of the operations).

The data concerning the proportion of dwellings of different types provided by the censuses are, however, subject to certain objections which should be examined. In the first place, the data relate to all the dwellings in the town in question, and not to the

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of a town or social class. In regard to food, each family consumes a fair number of articles, and while it may happen that certain families consume only certain qualities of bread or meat and other families consume other qualities, it may also happen that certain families consume both qualities in varying proportions. The divergence is, however, by no means fundamental, for when in constructing index numbers of food costs the consumption of an average family is considered—or of a " unit of consumption " in an average family—it is because this average family is taken as representing the whole of an urban population or social class.

[1] Strictly speaking it is the proportion of occupied dwellings that should be considered. Failing these figures, however, use may conveniently be made of the proportion of existing dwellings, since the proportion of unoccupied dwellings is as a rule too small to invalidate the figures concerning existing dwellings.

[2] Thus, assuming that the number of one-roomed and of five-roomed dwellings is equal, the rents of rooms belonging to five-roomed dwellings would be weighted five times more than those of rooms belonging to one-roomed dwellings.

workers' dwellings, which are the only ones that ought to be considered, since the question under consideration is that of the consumption habits of workers' families; and it is conceivable that the distribution of the types of dwellings may tend to differ substantially for the whole of an urban population and for the working class alone of the town in question.

Secondly, the distribution of dwellings by size is to some extent affected by the size of the families occupying them. To arrive at exact results, the "consumption habits" in regard to housing ought therefore to be expressed in terms of a family of a particular size, just as for the consumption of food they are expressed in terms of a " unit of consumption ".

There is not sufficient material to check closely the importance of these objections. A consideration of some of the data available, which are reproduced in tables II and III (pp. 122 and 123), however, gives an idea of the order of magnitude of the errors that may result from using these imperfect coefficients for calculating weighted averages of rents. Table II shows the effect of using weights relating to all dwellings instead of workers' dwellings only. Table III shows roughly the influence of the size of the family on the distribution of dwellings by size, and therefore on the average rent.

It will be seen that without being completely negligible, these objections do not appear to be sufficiently serious to exclude any attempt at combining the rents of dwellings of different types on this basis with a view to obtaining an average measure of the cost of housing. It will be remembered that in a general way, variations in the weights, provided they remain within certain limits, have only a comparatively small effect on the average. The errors affecting the weights considered here cannot therefore seriously disturb the significance of the averages, especially when account is taken of the fact that the actual rent figures cannot claim to be very exact.

Census data are also subject to another somewhat important reservation arising from the way in which the urban unit which is the basis for census statistics is fixed. This unit is generally an administrative unit and its limits rarely coincide with that of real area of the locality, either because it includes certain rural or semi-rural districts, or because it excludes certain residential districts outside the municipal boundaries or certain secondary areas which have gradually become in fact part of the adjacent municipality. The distribution of dwellings of different types

TABLE II. — DISTRIBUTION OF DWELLINGS BY SIZE FOR WORKERS' DWELLINGS AND ALL DWELLINGS IN CERTAIN TOWNS AND AVERAGE RENT PER ROOM WEIGHTED ACCORDING TO THESE TWO DISTRIBUTIONS

Size of dwellings (number of rooms)	Denmark (Copenhagen)		Norway (Oslo)			Switzerland (Zurich)		Czechoslovakia (Prague)		Irish Free State (Dublin)[1]	
	Workers' dwellings	All dwellings	Skilled workers' dwellings	Unskilled workers' dwellings	All dwellings	Workers' dwellings	All dwellings	Workers' dwellings	All dwellings	Workers' dwellings	All dwellings
	Percentage of dwellings of different sizes										
1	8.2	6.6	3.2	6.0	4.0	—	—	42.5	27.2	56.8	50.8
2	57.1	38.3	34.2	45.3	26.8	79.6	64.1	48.8	40.7		
3	24.2	23.5	42.9	36.2	30.6			6.9	17.4	28.8	27.9
4	9.2	17.5	13.4	9.2	14.5			1.4	9.2		
5	1.3	14.1	5.0	2.6	11.1	20.4	34.9	0.3	3.4	14.4	21.3
6 and over			1.3	0.7	13.0			0.1	2.2		
	Average rent per room (in dollars at gold parity) weighted according to the above distributions										
1-6 and over	3.13	3.29	3.43	3.38	3.49	5.11	5.15	4.06	3.93	3.01	3.00

[1] Percentages based on the proportion of persons (male) occupying dwellings of different size.

TABLE III. — DISTRIBUTION OF DWELLINGS BY SIZE FOR DWELLINGS OCCUPIED BY FOUR PERSONS AND FOR ALL DWELLINGS IN CERTAIN TOWNS AND AVERAGE RENT PER ROOM WEIGHTED ACCORDING TO THESE TWO DISTRIBUTIONS.

Size of dwellings (number of rooms)	Denmark (Copenhagen)		Finland (Helsinki)		Irish Free State (Dublin)		Norway (Oslo)		Poland (Warsaw)		Sweden (Stockholm)		Czechoslovakia (Prague)	
	4 persons	All dwellings	4 persons	All dwellings	4 persons	All dwellings	4 persons	All dwellings	4 persons	All dwellings	4 persons	All dwellings	4 persons	All dwellings
	Percentage of dwellings of different dizes													
1	2.1	7.2	38.2	41.2	26.3	31.4	1.3	4.0	42.8	39.1	4.8	14.2	24.1	29.4
2	33.0	38.1	31.7	28.4	19.8	20.3	25.0	26.8	26.0	25.2	39.3	34.9	45.3	43.2
3	25.3	23.1	12.4	11.0	13.4	13.2	31.9	30.6	16.1	17.6	27.0	24.1	17.8	15.6
4	21.1	17.2	8.1	7.3	11.8	11.3	15.8	14.5	7.9	8.3	12.0	10.6	12.3	10.7
5	9.1	6.9	4.2	4.4	8.1	7.5	13.2	11.1	4.1	4.7	6.2	5.5	0.4	0.7
6 and over	9.4	7.5	5.4	7.7	20.6	16.3	12.8	13.0	3.1	5.1	10.7	10.7	0.1	0.4
	Average rent per room (in dollars at gold parity) weighted according to the above distributions													
1 to 6 and over	3.36	3.74	—	—	3.04	3.08	3.51	3.49	2.72	2.74	9.48	10.12	3.92	3.93

SOURCES OF TABLES II AND III. — (a) *Percentage distribution of dwellings:*

Denmark: STATISTICAL DEPARTMENT: *Loyer et logement en novembre 1930*, p. 30 and pp. 132-133.
Finland: MINISTRY OF SOCIAL AFFAIRS: *Enquêtes spéciales sur les affaires sociales*, Vol. II: " Recensement des habitations du 25 avril 1919 ", pp. 110-111.
Irish Free State: DEPARTMENT OF INDUSTRY AND COMMERCE: *Census of Population, 1926*, Vol. IV: " Housing ", pp. 95 and 161.
Norway: CENTRAL STATISTICAL OFFICE: *Recensement de Norvège, 1er décembre 1920*, vol. VII: " Statistiques d'habitations ", pp. 87 and 200.
Poland: CENTRAL STATISTICAL OFFICE: *Le premier recensement général*, 30 Sept. 1921, vol. XIV, p. 3.
Sweden: MUNICIPALITY OF STOCKHOLM: *Le dixième recensement général des habitations de Stockholm*, 31 Dec. 1920, p. 67.
Switzerland: FEDERAL STATISTICAL OFFICE: *Résultats de l'enquête fédérale sur les logements au 1er décembre 1920*, p. 148.
Czechoslovakia: STATISTICAL COMMITTEE OF PRAGUE: *Statistique des logements à Prague*, 15 Feb. 1921, pp. 114-115 and 349.

(b) *Rents*: Data taken from table IV.

within the administrative boundaries may therefore be very different from what it would be if the whole of these adjoining areas were included. This fact is obviously of some importance and it will be necessary, when definite rent index numbers are compiled, to examine in each case the character of the administrative units used in the Census reports and to make, if necessary, with the help of competent national authorities, the appropriate readjustments.

Another disadvantage of census data is that they rarely distinguish between dwellings with and without a bathroom. This is clearly a defect that must be remedied in subsequent enquiries, if necessary by way of approximation, if it is sought to combine the rents of dwellings not only of different sizes, but also of the two degrees of comfort [1].

METHODS OF CALCULATION

The basis of weighting having thus been determined, we still have to examine the method of weighting most suited to the purpose of the index numbers, which is to give a measure of the relative level of rents paid in different towns for " a unit of housing ", account being taken to a greater or lesser extent of the special conditions of each locality. It is understood that at a later stage of the researches the question of the quantity of these housing units to be included in the budget or budgets will be discussed: that is to say, the weight to be assigned to the index of rents so obtained when combining it with indexes for other groups.

It is this latitude to allow " to a greater or lesser extent " for the special conditions of each locality that makes it possible to choose between various methods of calculation. The choice, it must be admitted, is in some degree arbitrary. It depends on the point of view adopted and the importance it is proposed to attach to one factor rather than to another.

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[1] It is equally unnecessary to be held up by the fact, previously mentioned in the article in the *Review*, that in certain countries the data refer not to dwellings as architectural units (that is to say, the aggregate of rooms intended for occupation by a household), but to dwellings as social units, that is to say, the aggregate of rooms actually occupied by a household. This difference in the conception of the dwelling on which the statistics are based obviously produces a difference in the figures. It is possible that when the final index numbers are compiled, an adjustment of the data should be made to take account of this fact. For the present, during this purely experimental stage of the researches, they can perhaps be used as they stand.

Only some of the possible methods are considered here taking into account both the general theoretical line adopted for the work so far done by the Office and the practical possibilities afforded by the material that has been collected or may reasonably be expected to be obtained from future enquiries.

Method I: Variable Weights

The method that would best allow for the " consumption habits " of each town would obviously be one of variable weighting. The average rents would be determined for each town by the use of weights that would differ in each case, giving expression to the conditions peculiar to each locality. An index would then be calculated from these average rents.

This is the method which also corresponds most closely to the operation of determining the average price of any particular article in the food group. The average price of bread, for instance, is obtained by taking the average of the prices recorded in different districts, in different types of shop, and possibly—as in the hypothesis made above—for different qualities. If the prices so recorded show marked divergences, the average will have to be weighted at least approximately according to the consumption of bread sold in the different districts or shops or belonging to the different qualities in the town in question.

This method has the advantage of being comparatively easy to carry out. To obtain a satisfactory average of rents it is in fact sufficient to have data of the rents of the predominant types of dwellings in each town. The figures concerning the less usual types are not indispensable, since they do not seriously affect the average. It is not even necessary to require that the national statistics should give separate data for dwellings with and without a bathroom, provided that each category of comfort is suitably represented in the figures relating to the different categories of size. If this method were to be adopted for constructing international rent index numbers, the periodical collection of rent data might be much simplified, so that the comparisons could probably be made for a fairly large number of countries.

The method suffers, however, from the disadvantage of making the average refer to types of dwellings which may differ from one town to another, or at least of assigning a different importance to each type. Thus it can never be known whether the difference in the average rent observed is due to this divergence in the composition of the average or to an actual difference in the rents of corresponding dwellings.

It may be added that if the method of variable weights is simplified to the utmost, it might possibly be sufficient to take for each town, not the average of the rents of certain of the more important types of dwellings, but simply the rent of the predominant type of dwelling or, in statistical terms, the mode. The practical value of such a procedure, which has the advantage of great simplicity, will be discussed below.

Method II: Standard Weights

The disadvantages of the method of variable weights (or of the mode) are avoided by a second method that can be contemplated here, that of standard weights in which a standard list of weights is applied uniformly to all towns. From the theoretical point of view this method is somewhat open to question, especially if the comparisons are to relate to towns with very different housing conditions. The standard weights are in that case especially difficult to determine, and assuming that an average is struck between the conditions of the different towns, these average weights will necessarily differ markedly from the actual facts in certain countries, so that there is a risk that the average rents obtained are somewhat fictitious.

From the practical point of view the method also involves difficulties. If a somewhat complete measure of the level of rents is sought, the averages must cover all types of dwellings of any importance in all the towns considered. In view of the differences between towns in regard to the predominance of large and small dwellings, of dwellings with and without a bathroom, all the twelve categories of size and comfort would have to be covered completely for all the towns. To judge from the results obtained by the first enquiry, there can be no doubt that great obstacles would have to be overcome in collecting such extensive material.

Method III: Double Weights

It may therefore be useful to consider a third method, no doubt more refined, but avoiding certain of the theoretical and practical disadvantages of the methods of standard and variable weights. This is the method of double weights inspired by Fisher's " ideal " formula, which has been used by the Office for its researches into comparative food costs.

In this case the comparisons are made for pairs of towns (with a possibility of always including one particular town in each pair so as to have a general basis common to all the comparisons). A double index number is calculated for each pair, one with the weights corresponding to the conditions prevailing in one of the towns, the other with the weights corresponding to the conditions in the other town, and the average is then taken of the two indexes. The indexes of each pair of towns are in this way determined with as much certainty and as little fictitiousness as possible. They are not necessarily based on the same list of categories of dwellings, so that a certain measure of adjustment to the available data is possible. Only for the basic town will it be necessary to have data for a large number of categories of dwellings, possibly for all twelve. The principal practical disadvantages of the method, which has great advantages from the theoretical point of view, is that the calculations involved are somewhat laborious.

Method IV: Mixed System

It is therefore possible that a more practicable solution will lie in a mixed system combining the second and third methods. Standard weights might be used without raising serious theoretical objections or practical difficulties for comparisons limited to towns where the housing conditions are comparatively alike. Different series of index numbers might thus be compiled, each with the standard weights corresponding to the conditions in the towns it covers. The indexes for the base towns of each of these independent series would then be linked up by the more refined method of double weighting.

This mixed system—inspired by that suggested for comparisons of food costs—obviously raises the problem of determining " regions of good comparability ", within which the indexes would be based on a simple system of standard weights. Does this mean that for determining these " regions " the procedure to be adopted must be as elaborate as that contemplated for the comparisons of food costs ? This is a question that may be considered later, for theoretically at least it is conceivable that the problem of the international comparison of housing costs should be studied on the same lines as that of the comparison of food costs. At the present stage of research and in the present state of available material, a much rougher method would seem to be sufficient. A simple comparison of the percentages expressing the importance of the different types of dwelling in the different towns will no doubt be adequate for determining the region of " good comparability ".

It is clearly a somewhat difficult matter to decide between the various advantages and disadvantages of the methods of weighting mentioned. Instead of going into the matter more thoroughly from the theoretical standpoint, useful light can be thrown on the problem by making some attempts at practical application. They will show to what extent the results obtained by the various methods do in fact differ, and at the same time bring out the practical difficulties to be met with in using the available statistical material, or even material that may be expected to become slightly improved.

III

PRACTICAL APPLICATION

The Statistical Material Used

For making our experimental calculations some fifteen towns have been selected from among the somewhat heterogeneous material collected for the enquiry of October 1932, belonging to different countries, for which the information was most complete. These figures are reproduced in table IV below, after having been subjected to certain treatment.

The rent data (including subsidiary payments and taxes) are expressed per room and in a common currency, the dollar, calculated at gold parity[1]. The data are classified according to kind, that is to say according as they relate to workers' dwellings or all dwellings, for it is clear that comparisons can be made only between towns which give figures of the same kind. Similarly, for each series of data a distinction has been made between those relating solely to dwellings without a bathroom and those relating to dwellings with or without a bathroom[2]. In this respect, too, the comparisons must be limited to groups of towns for which the data are similar from the point of view of the presence or absence of a bathroom.

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[1] The dollar at gold parity has been chosen here so as to allow of combining the rent indexes obtained with the indexes of food prices calculated elsewhere on this basis. The gold rate of exchange is obviously quite theoretical, but can hardly be objected to here in view of the experimental nature of the present calculations.

[2] This latter classification has led to certain difficulties for the Swedish towns, for which the figures do not distinguish between dwellings according to the presence or absence of a bathroom, but according to the presence or absence of central heating. In view of the fact, however, that the presence of a bathroom has been taken as a criterion of the degree of general comfort of the dwelling, it seems that for a country with the climate of Sweden the criterion of central heating would be just as indicative (especially as the Office was informed that most dwellings equipped with central heating would also have a bathroom). This criterion of central heating has therefore been treated on the same footing as that of a bathroom.

TABLE IV. — MONTHLY RENT PER ROOM OF DWELLINGS OF DIFFERENT TYPES IN 1932 (IN DOLLARS AT GOLD PARITY)

Town	Number of rooms					
	1	2	3	4	5	6
	1. *Workers' dwellings:*					
	(a) *without bathroom*					
Utrecht	3.27	2.40	2.18	1.88	1.94	(2.00)
Birmingham	—	(2.50)	2.43	2.25	2.09	(2.09)
Dublin	3.87	2.42	2.22	3.28	2.94	(3.00)
Oslo	—	3.20	3.48	3.72	3.89	(4.00)
	(b) *with or without bathroom*					
Copenhagen	(3.00)	2.94	3.45	3.51	3.33	4.41
London	—	4.99	4.22	3.84	3.58	—
Budapest	3.85	2.98	4.26	5.29	6.30	(6.80)
	2. *All dwellings:*					
	(a) *without bathroom*					
Stockholm [1]	9.11	6.84	7.41	8.18	8.52	(8.65)
Tallinn	4.52	2.90	2.83	2.87	(2.90)	—
Riga	—	1.51	2.11	2.30	(2.40)	—
Warsaw	2.79	2.32	2.59	2.99	3.54	(3.90)
Zurich	—	(4.33)	4.33	4.43	4.56	—
Göteborg [1]	5.90	4.83	5.72	6.37	8.32	(8.80)
Malmö [1]	4.82	4.16	4.38	4.96	5.36	(5.60)
	(b) *with or without bathroom*					
Stockholm	15.20	8.82	9.02	9.91	10.45	—
Zurich	—	(5.01)	5.01	5.19	5.42	—
Oslo	—	3.48	3.84	4.31	5.15	—
Göteborg	7.16	5.39	6.56	7.21	8.55	—
Malmö	5.53	4.19	4.55	5.25	5.88	—

[1] Rents of dwellings of a corresponding degree of comfort.

Note. — Figures in brackets are estimates.

The material so collected and classified suffers, however, from a numbers of gaps, which seriously limit the possibility of applying the proposed methods experimentally, in particular the second and third. It has therefore had to be completed by estimates [1]. These, it is thought, could reasonably be made by taking into account the curve of variations from category to category (which,

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[1] Certain calculations have also been made so as to include a larger number of towns in the group of data concerning dwellings with or without a bathroom.

TABLE V. — INDEX NUMBERS OF RENTS PER ROOM (IN DOLLARS AT GOLD PARITY) OF DWELLINGS OF DIFFERENT TYPES IN 1932

Town	Number of rooms				
	1	2	3	4	5
			1. *Workers' dwellings:*		
			(a) *without bathroom*		
Utrecht	100	100	100	100	100
Birmingham	—	—	111.5	119.7	107.7
Dublin	118.3	100.8	101.8	174.5	151.5
Oslo	—	133.3	159.6	197.9	200.5
			(b) *with or without bathroom*		
Copenhagen	—	100	100	100	100
London	—	169.7	122.3	109.4	107.5
Budapest	—	101.3	123.5	150.7	189.2
			2. *All dwellings:*		
			(a) *without bathroom*		
Stockholm[1]	100	100	100	100	100
Tallinn	49.6	42.4	38.2	35.1	—
Riga	—	22.1	28.5	28.1	—
Warsaw	30.6	33.9	35.0	36.6	41.5
Zurich	—	—	58.4	54.2	53.5
Göteborg[1]	64.8	70.6	77.2	77.9	97.7
Malmö[1]	52.9	60.8	59.1	60.6	62.9
			(b) *with or without bathroom*		
Stockholm	100	100	100	100	100
Zurich	—	—	55.5	52.4	51.9
Oslo	—	39.5	42.6	43.5	49.3
Göteborg	47.1	61.1	72.7	72.8	81.8
Malmö	36.4	47.5	50.4	53.0	56.3

[1] Rents of dwellings of a corresponding degree of comfort.

as indicated above, is as a rule quite characteristic), or simply by reproducing the figure for the neighbouring category when the tendency is not clear. In fact, the estimates have been limited to one or two types of dwelling for each town; they are given in brackets in table IV.

The weights chosen are the percentages of dwellings of different sizes, calculated from the census data that were used for the 1932 enquiry. They are reproduced below in table IX.

Experimental Calculations

With this material, adjusted and completed in the manner indicated, the various methods of weighting considered above have been applied by way of experiment; viz:

Method I: Variable weights;
Method II: Standard weights;
Method III: Double weights;
Method IV: Mixed system combining methods II and III.

The indexes obtained will of course be very fragmentary, since in the present state of the material comparisons are possible only between towns for which the available data are of the same kind —workers' dwellings and all dwellings—or relate either to dwellings without a bathroom or to dwellings of whatever degree of comfort.

Further, the various methods of weighting can be applied only according to the criterion of the size of the dwellings, and, in addition, the averages cannot be made to cover all the sizes in each town, since the statistical material is not yet complete enough for that—even with the estimates we have made. The scope of the averages has therefore been limited as follows: for method I (variable weights) the average rents have been calculated for the three principal sizes in each town, representing about 65 to 80 per cent. of all the dwellings (the estimated figures not being used); for the methods II and III (standard weights and double weights) the scope is uniformly limited to the four central sizes (dwellings of two to five rooms), but the estimates are taken into account (not more than one for each town).

The results of these calculations are given in tables VI and VII. Table VI shows the average rents obtained and table VII the index numbers calculated from the average rents, the base being one of the towns in each group of data. The tables also show the " mode " of the rents and the index calculated from the mode, to which reference was made in the theoretical discussion on the method of variable weights.

For a few towns, however, it was possible to calculate more comprehensive averages, greater use being made where necessary of the estimated figures. The methods I and II of weighting have been applied with this wider scope so as to bring out the difference

TABLE VI. — AVERAGE RENTS CALCULATED ACCORDING TO DIFFERENT METHODS OF WEIGHTING

Town	Mode	System of weights							
		I Variable	II Standard	III Double		IV Mixed: according to coefficient of " regions " where the number of rooms in the predominant type of dwelling is:			
				A	*B*	1	2	3	4
1. *Workers' dwellings:* (a) *without bathroom*									
Utrecht[1] .	(2.00)	1.97	2.13	2.04	1.99-2.17[3]	—	—	—	—
Berlin . . .	3.43	—	—	—	—	—	—	—	—
Birmingham	2.09	2.23	2.35[1]	2.27	2.23	—	—	—	—
Glasgow	3.96	—	—	—	—	—	—	—	—
Dublin	3.87	3.07	2.66	2.83	2.63	—	—	—	—
Oslo	3.48	3.42	3.52	3.65	3.48	—	—	—	—
(b) *with or without bathroom*									
Copenhagen	3.45	3.44	3.29	3.40	3.19-3.34[4]	—	—	—	—
London	3.71	4.27	4.25	4.04	4.19	—	—	—	—
Budapest	2.98	3.73	4.47	4.86	3.88	—	—	—	—
2. *All dwellings:* (a) *without bathroom*									
Stockholm[2]	6.84	7.38	7.61	7.30	7.12-8.12[5]	(8.11)	7.27	—	(8.00)
Tallinn	—	3.58	2.87	2.87	2.88	3.56	(2.88)	—	—
Riga	1.51	1.79	2.02	1.86	1.84	—	1.82	—	—
Warsaw	2.79	2.60	2.77	2.57	2.61	2.70	(2.55)	—	—
Zurich[2]	4.33	4.42	4.39	4.36	4.45	—	(4.56)	—	4.44
Göteborg[2]	4.83	5.13	6.06	5.53	5.25	—	5.47	—	—
Malmö[2]	4.16	4.32	4.62	4.41	4.37	—	4.39	—	—
(b) *with or without bathroom*									
Stockholm	8.82	10.28	9.42	9.14	9.06-9.89[6]	—	9.11	(9.37)	(9.78)
Oslo	3.84	3.79	4.07	3.82	3.96	—	(3.78)	4.05	—
Zurich	5.01	5.22	5.13	5.06	5.23	—	(5.06)	—	5.20
Göteborg	5.39	5.86	6.71	6.23	6.01	—	6.13	—	—
Malmö	4.19	4.44	4.84	4.56	4.54	—	4.52	—	—

A = Average rents weighted by the coefficient for the base town.
B = Average rents weighted by the coefficient for the other town of the pair.

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[1] The average includes an estimate for the rents of six-roomed dwellings; the mode is in fact given by this (estimated) category.

[2] The weighting is based on the proportion of dwellings of different sizes in the category of comfort considered and not for all dwellings as is the case for other towns.

[3] Utrecht rents weighted by the coefficients for Birmingham: 1.99; for Dublin and Oslo: 2.17.

[4] Copenhagen rents weighted by the coefficients for London: 3.34; for Budapest: 3.19.

[5] Stockholm rents weighted by the coefficients for Tallinn: 7.26; Riga: 7.30; Warsaw: 7.36; Zurich: 8.12; Göteborg: 7.12; Malmö: 7.24.

[6] Stockholm rents weighted by the coefficients for Oslo : 9.28; Zurich : 9.89; Göteborg : 9.06; Malmö: 9.12.

TABLE VII. — INDEX NUMBERS OF RENTS CALCULATED ACCORDING TO DIFFERENT METHODS OF WEIGHTING

Town	Rent index numbers calculated according to:								Index numbers of retail food prices
	Mode	System of weights							
		I Variable	II Standard	III Double			IV Mixed		
				A	*B*	Final index	Regional index	Inter-regional index	
	1. *Workers' dwellings* [1]: (a) *without bathroom*								
Utrecht	100.0	100.0	100.0	100.0	100.0	100.0	—	—	100.0 [1]
Birmingham	104.5	113.2	110.3	111.3	112.1	111.7	—	—	125.2
Dublin	193.5	155.8	124.9	138.7	121.2	129.7	—	—	130.9
Oslo	174.0	173.6	165.3	178.4	160.4	169.2	—	—	117.1
	(b) *with or without bathroom*								
Copenhagen	100.0	100.0	100.0	100.0	100.0	100.0	—	—	100.0
London	107.5	124.1	129.2	118.8	125.4	122.1	—	—	115.0
Budapest	86.4	108.4	135.9	142.9	121.6	131.8	—	—	86.6
	2. *All dwellings* [1]: (a) *without bathroom*								
Stockholm	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Tallinn	—	48.5	37.7	39.3	39.7	39.5	—	41.7	41.7
Riga	22.1	24.3	26.5	25.5	25.2	25.3	25.0	—	52.6
Warsaw	40.8	35.2	36.4	35.2	35.5	35.3	—	34.2	44.0
Zurich	63.3	59.9	57.7	59.7	54.8	57.2	—	59.0	86.3
Göteborg	70.6	69.5	79.6	75.8	73.7	74.7	75.2	—	95.4
Malmö	60.8	58.5	60.7	60.4	60.4	60.4	60.4	—	99.2 [1]
	(b) *with or without bathroom*								
Stockholm	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Oslo	43.5	36.9	43.2	41.8	42.1	42.2	—	42.3	86.3
Zurich	56.8	50.8	54.5	55.4	52.9	54.1	—	54.3	82.5 [1]
Göteborg	61.1	57.0	71.2	68.2	66.3	67.2	67.3	—	95.4
Malmö	47.5	43.2	51.4	49.9	49.8	49.8	49.6	—	99.2

A = Average rents weighted by the coefficient for the base town.
B = Average rents weighted by the coefficient for the other town of the pair.

[1] Price for the average of several towns in the country considered.

that may result when the scope of the averages differs [1]. For the method I (variable weights) four or five sizes of dwellings, representing 80 to 85 per cent of the total number, have been taken and for the method II (standard weights), six sizes. The average rents so obtained are given in table VIII and compared with those calculated on the narrower basis of table VI.

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[1] Similar calculations have been made by the method of double weights, but the divergences due to the difference of scope are all of the same order of magnitude, and it has not been thought necessary to reproduce the figures here.

The application of the method II (standard weights) also means calculating standard coefficients. In principle, this ought to necessitate the preliminary determination of the geographical field of investigation, since they are deemed to give expression to the average conditions prevailing in all the towns covered. Owing to the experimental nature of the calculations, which are arbitrarily limited to certain towns for which data are available, it was believed that these coefficients could be fixed more or less roughly. We have therefore simply taken the arithmetic mean of the coefficients of the twelve towns for which satisfactory data were available, representing more or less fairly the various geographical regions (with different " consumption habits ") covered by our calculations [1].

The application of the method IV (mixed system combining methods II and III) is not of much interest here, since the means it offers of simplifying the calculations are very small, owing to the small number of towns between which indexes can be calculated.

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[1] The twelve towns are the following: Birmingham, Dublin, Montreal, representing English-speaking countries (European and overseas); Stockholm and Oslo, representing Northern Europe; Warsaw, Tallinn, Riga and Budapest, representing Eastern and East Central Europe; Zurich, representing Central Europe; Utrecht and Copenhagen, representing North Eastern Europe. It will be seen that for lack of proper data the greater capitals, London, Paris, and Berlin, are not included. Further, important countries such as France, Italy and Germany, as also Czechoslovakia and Austria, are not directly represented. The absence of the two latter countries is compensated for by a somewhat excessive importance attached to the countries of Eastern Europe, where the distribution of the different categories is similar. The position is the same for Germany, which is fairly well represented by the figures for the three small neighbouring countries, Switzerland, the Netherlands and Denmark. On the other hand, there is a complete gap for Western Europe. The standard coefficients obtained are as follows:

Size of dwellings (number of rooms)	Percentage
1	10.7
2	21.4
3	19.5
4	16.4
5	12.3
6 and over	19.7

This distribution, resulting from an average of very different conditions, appears fairly normal. The average-sized dwellings preponderate but not too markedly and the extreme categories are still reasonably represented. The proportion of dwellings of six rooms and over is perhaps rather high.

It has been thought useful, however, to consider the practical aspects of the problem of determining regions of good comparability.

TABLE VIII. — AVERAGE RENTS OF MORE AND LESS RESTRICTED SCOPE ACCORDING TO DIFFERENT METHODS OF WEIGHTING

Town	I Variable weights		II Standard weights	
	Scope of average			
	Narrow	Wide	Narrow	Wide
	Workers' dwellings: (a) *without bathroom*			
Utrecht . . .	1.97	2.04	2.13	2.23
Dublin	3.07	3.09	2.66	2.86
Oslo	3.42	3.48	—	—
	(b) *with or without bathroom*			
Copenhagen . .	3.44	3.54	3.29	3.48
London . . .	4.27	4.19	—	—
Budapest . . .	3.73	3.87	4.47	4.86
	All dwellings: without bathroom			
Stockholm . .	7.38	7.54	7.61	7.98
Tallinn . . .	3.58	3.51	—	—
Warsaw . . .	2.60	2.68	2.77	2.99
Göteborg . .	5.13	5.31	6.06	6.58
Malmö	4.32	4.40	4.62	4.84

In the theoretical discussion of the question mention was made of a rough method, consisting in a simple comparison of the distribution of dwellings by size in the different towns and on the assumption that those towns which have the same predominating type of size belong to a region of good comparability. This has been done in table IX, which also includes certain towns hitherto left out of the calculations, for the purpose of obtaining a better idea of the possibilities of the method.

The figures in this table are, however, only given as an indication, since, as has been already pointed out, the classification might be somewhat different if the boundaries of the administrative units on which the census statistics are based corresponded more exactly to the actual inhabited areas.

TABLE IX. — DISTRIBUTION OF DWELLINGS BY SIZE IN DIFFERENT TOWNS CLASSIFIED ACCORDING TO PREDOMINANT TYPE OF DWELLING

Town	Percentage of dwellings of					
	1 room	2 rooms	3 rooms	4 rooms	5 rooms	6 rooms and over
	Predominant type of dwelling: 1 room					
Warsaw[1]	39.1	25.2	17.6	8.3	4.7	5.1
Tallinn[1]	70.7[2]		11.6	8.0	4.4	5.3
Dublin[1]	31.4	20.3	13.2	11.3	7.5	16.3
	Predominant type of dwelling: 2 rooms					
Vienna[1]	14.2	55.2	14.4	9.2	7.0	
Göteborg	6.1	51.9	20.2	8.1	4.6	7.1
Prague (controlled rents)	26.2	44.1	17.0	8.9	2.6	1.2
Prague (uncontrolled rents)	28.7	37.3	19.7	9.0	3.2	2.1
Budapest[1]	0.2	48.7	29.4	13.1	5.7	2.9
Riga[1]	3.6	47.1	21.9	12.6	6.8	8.0
Glasgow[1]	14.5	43.6	24.0	9.0	4.2	4.7
Malmö	6.8	41.4	28.6	9.1	5.9	8.2
Stockholm[1]	16.2	34.3	23.9	9.4	5.1	11.1
Cork	12.5	23.2	18.4	18.6	11.0	16.3
	Predominant type of dwelling: 3 rooms					
Basle[1]	4.1	29.7	40.1	12.6	7.3	6.2
Berlin	4.4	29.6	35.4	25.5[3]		5.1[4]
Oslo[1]	0.0	29.1	32.2	14.4	10.7	13.6
Breslau	11.1	27.2	28.7	28.2[3]		4.8[4]
Munich[1]	6.2	24.0	25.3	38.2[3]		6.3[4]
Copenhagen[1]	0.0	6.4	38.4	23.8	17.4	14.0
Hamburg	0.4	3.7	31.5	56.7[3]		7.7[4]
Cologne	3.6	19.4	27.5	41.1[3]		8.4
	Predominant type of dwelling: 4 rooms					
Dundalk[1]	2.4	23.2	13.5	31.0	11.8	18.1
Leeds[1]	0.4	11.3	23.8	29.3	17.8	17.4
Newcastle	2.0	16.4	27.9	28.3	11.7	13.7
Zurich[1]	0.0	1.7	13.8	44.1	22.5	17.9
Manchester[1]	0.2	1.8	10.7	41.3	26.4	19.6
	Predominant type of dwelling: 5 rooms					
Birmingham[1]	0.2	1.1	17.7	20.2	30.5	30.3
Bristol[1]	0.3	1.5	3.7	18.8	36.5	39.2
	Predominant type of dwelling: 6 rooms					
Ottawa	0.2	0.9	1.6	4.8	8.1	84.4
Toronto	0.2	0.8	3.4	6.1	9.6	79.9
Halifax (Canada)	0.4	1.6	3.6	12.9	11.4	70.1
Winnipeg	0.3	1.1	6.0	7.9	20.2	64.5
Vancouver	1.1	3.3	5.6	11.9	15.8	62.3
Utrecht[1]	1.9	6.4	7.7	12.8	13.4	57.8
Montreal[1]	0.2	1.4	7.0	18.8	18.4	54.2

[1] Towns taken into consideration for calculating the regional standard coefficients.
[2] Estimate: one room, 42.7 per cent; 2 rooms, 29.0 per cent.
[3] Dwellings of 4 to 6 rooms.
[4] Dwellings of 7 rooms and over.

It will be seen that the "regions of good comparability" thus obtained do not always correspond to geographical "neighbourhood", and that they contain towns very distant from each other in which the predominant type of dwellings is the same. In other cases certain towns of one and the same country are classified in different regions, and this raises a more difficult problem [1]. It should also be noted that within each region there are fairly marked differences in the distribution of dwellings by size. The predominant types predominate only to varying extent, and the distribution of the other types around the predominant type also varies.

This rough method, therefore, does not seem altogether satisfactory, and will no doubt have to be completed by a more thorough examination of the situation in each town when it becomes a question of compiling the final index numbers of rents according to regions of good comparability. For the experimental calculations attempted here this basis seems sufficient, however, and we have determined regional standard coefficients on the same lines as the general standard coefficients obtained by method II, that is to say, by taking the simple arithmetic mean of the percentages of certain towns (chosen this time in each region) [2].

With the help of these figures the mixed method of calculating index numbers has been applied to a small number of towns for which the available data lent themselves best to illustrating this procedure. The scope of the averages has been limited to the four

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[1] It may also be that a certain overlapping of the national frontiers may be due to the difficulties already noticed in the census statistics which are based on administrative units. This appears particularly to be the case for certain English towns.

[2] The towns selected for establishing the averages are given in note 1 of table IX. The same twelve towns were selected for determining the standard coefficients for method II, together with the following towns: Vienna, Glasgow, Basle, Munich, Dundalk, Manchester, so as to represent each region adequately. The coefficients obtained are the following:

Size of dwellings (Number of rooms)	Standard weights for regions of good comparability where the predominant type of dwelling has:					
	1 room	2 rooms	3 rooms	4 rooms	5 rooms	6 rooms
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1	38.1	9.7	2.6	0.9	0.3	1.1
2	24.2	45.8	22.3	8.9	1.3	3.9
3	14.1	22.7	34.0	12.7	10.7	7.3
4	9.2	10.7	17.5	38.8	19.5	15.8
5	5.5	5.1	13.6	20.2	33.5	15.9
6	8.9	6.0	10.0	18.5	34.7	56.0

central sizes of dwellings, except for towns where the predominant type of dwelling is that of one room, so that this size has also been taken into account. The results of the calculations are also given in tables VI and VII.

EXAMINATION OF RESULTS

In comparing the results obtained by these different methods, we felt that attention should first be directed to the average rents obtained for calculating the index numbers, since the indexes themselves are biased by the average rent of the base town, here chosen arbitrarily and subject to possibly accidental conditions.

Before making these comparisons it is, however, necessary to obtain at least an approximate idea of the margin of error affecting these averages, owing to certain imperfections that could not be avoided when calculating them. The main imperfections, it may be remembered, are of two kinds; those inherent in the weights, which relate to all the dwellings of the town in question, whereas they ought to reflect the housing conditions of workers' families only, account being taken of differences in the size of family; and the imperfections in the scope of the averages, which, for lack of data, could not as a rule be obtained for all the sizes of dwellings. On the first point tables II and III give some information. They show that the deviations due to the use of more or less satisfactory weights are generally slight; in some cases, however, they amount to as much as 5 or 6 per cent. and even 10 per cent. On the second point, table VIII shows that the average rents obtained by the same method, but with a difference in scope, also differ very little, the deviations being of much the same order of magnitude as those resulting from imperfection in the weights; on the other had, they display a fairly definite bias, from which the others do not suffer: the wider averages are nearly always higher than the narrow averages. This is clearly due to the fact that the extreme categories are eliminated when the scope of the average is restricted, that is to say, those for which the rents per room are in most cases the highest.

While these observations certainly cannot be given general validity, they yet give some information on the order of magnitude of the divergences to which attention should be paid in comparing the average rents obtained by the various methods of weighting.

In comparing the average weights obtained by method I, II and III (table VI), it will be seen that as a whole the divergences are not very great; they usually do not exceed 10 per cent., and in many cases are under 5 per cent. In certain special cases, however, they are higher. In particular, they are fairly substantial (in some cases up to 15-25 per cent.) between the mode and the other averages. This would seem to suggest that the distribution of rents by size is neither regular nor gradual enough to allow of taking the mode as an adequate approximation to an average representing the special conditions of each country [1].

Between the average rents of methods I and II, the deviations are widest precisely for towns where the distribution of dwellings differs most from the standard distribution adopted as the basis of weighting for method II. This brings out the fictitious element in the averages of method II and the extent to which this fictitious element varies according to the town.

To compare the results of method III, account must be taken of the two average rents established for each town, on the one hand with the weights for the base town (*A* rents), and on the other with the weights of the other town of the pair (*B* rents).

It should be noted that the *A* rents of the base towns and the *B* rents of the towns that are not base towns should be identical with those obtained by method I, since in the two cases the weighting gives expression to the special conditions of each town. The slight differences observed are due to a difference in the scope of the averages [2].

On the other hand, for the *A* rents of the towns other than the base towns, and the *B* rents of the base towns, where the weights used are those of other towns than those considered, the averages obtained are more or less fictitious. This fictitious element is brought out by a comparison with the averages of method I (account being taken of the small irregularities due to differences in scope which also occur here). Taken as a whole the divergences are not so great as those observed between methods I and II, which are similarly due to the more or less fictitious character of the averages of method II. But in the case of method III these divergences are less of a drawback, since the average rents are used only to calculate one of the two partial indexes forming a final index.

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[1] It is true that the divergences are almost as large when the rents of method I are compared with some of the rents of method III, namely those compiled by using weights of another town than the one considered. But here the situation is different, for the *B* rents form only one of the two measures used for constructing the index, whereas the mode might be taken as the final measure.

[2] In the few cases in which they are somewhat considerable (Dublin, Stockholm), this is due to the fact that the rent of the size of dwellings which are covered or not covered by the averages differs widely from those of the other sizes.

In examining the results of method IV (mixed system), a distinction must be made between the average rents obtained with the standard weights of the region to which the town belongs (which must be used for calculating the regional indexes), and those based on the weights for a region other than that to which the town belongs (which must be used for one of the partial indexes of the inter-regional indexes obtained by the system of double weights). Thus the following comparisons are to be made.

(*a*) Comparison of the average rents by the method IV weighted according to conditions in the region to which each town belongs (figures not in brackets in the table VI) with the equivalent averages of method I and method III (*A* rents of the base and *B* rents of towns other than the base town): the deviations are insignificant and, taken as a whole, definitely smaller than those shown by a comparison of method I with method II. The somewhat larger divergences found for certain towns are due to differences in the scope of the averages compared [1].

(*b*) Comparison of the rents of method IV weighted according to a region other than that to which the towns belong (figures in brackets) with the " representative " averages of method I: the divergences are fairly substantial, being due above all to the more or less fictitious nature of the rents obtained by the mixed method [2]. But this fictitious character appears no more marked than was the case with the averages of method III. (*B* rents of the base towns and *A* rents of the towns that are not the base towns.)

This last comparison therefore leads to a fairly favourable conclusion for the mixed method, so far as the limited and fragmentary material available allows of any conclusion at all. Subject to this reservation, it appears in fact that on the one hand the average rents calculated (for establishing regional indexes) with the regional standard weights give results that are nearly as " representative " as those of method I, which are weighted according to the special conditions of each town. On the other hand, the rents weighted by the coefficients for other regions than those of the town considered (for the calculation of the partial indexes for the inter-regional indexes) are no more fictitious than those obtained by applying method III, using the coefficients of the other town of the pair.

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[1] In the comparison with method I for the towns of Göteborg (dwellings without bathroom), Stockholm and Malmö (dwellings with bathroom), the averages of method I cover one-roomed dwellings where the rents are particularly high compared with other types of dwellings, whereas the averages of the mixed method are limited to the four central sizes of dwellings. In the comparison with method III for the towns of Dublin and Tallinn, it is the averages of the mixed method which comprise the group of one-roomed dwellings where the rents are particularly high, while the averages of method III are limited to the four central sizes of dwellings.

[2] As a matter of fact, in these cases the scope of the averages is as a rule the same.

If, finally, we compare the actual indexes (table VII), and not the average rents used for calculating the indexes, it is to be expected, for the reason already given, that the divergences may be different from those shown by a comparison of the average rents. The index of modal figures differs substantially in certain cases from the other indexes. The divergences between indexes I and II are also considerable for certain towns (e.g. Dublin, Budapest). Although these cases are few for the towns covered by our experimental calculations, their existence shows that each method gives results that differ markedly as soon as the conditions become substantially different. In other towns the results, it is true, are very close, but these are towns where the conditions are similar to the standard adopted for weighting index II. These observations, combined with those resulting from the comparisons of the average rents, suggest that it would hardly be desirable to adopt simply the index of the modal figure, or indexes weighted according to methods I and II, as a general measure of the relation between rent levels. Each of these indexes reflects the phenomenon from a different angle, and the results do not coincide sufficiently to allow of accepting only one.

The practical application of the index weighted according to method III gives the results that might be expected from the theoretical character of the method. In every case considered but one, the index lies in fact between those weighted according to methods I and II, and this would seem to confirm the conclusion that it offers a more general and reliable measure. It might therefore be desirable to advocate its adoption, were it not that it suffered from the disadvantage of calling for somewhat laborious calculations and particularly complete statistics, the regular collection of which would be difficult to realise in practice. The method IV conceived as a means of overcoming this practical difficulty without too much affecting the theoretical value of the index, may probably be regarded as the solution to be adopted ultimately. But the difficulties met with in applying it in practice to the material at our disposal suggests that it is not yet possible to accept it finally. It is true that this mixed index is close to that obtained by method III, both in its regional indexes calculated from the regional standard coefficients, and in its inter-regional indexes calculated by the system of regional double weights. But both sets of indexes could not be applied in a large enough number of cases to allow us to regard the experiment as conclusive.

IV

CONCLUSIONS

The preceding analysis clearly cannot be expected to lead to any final conclusions. The theoretical examination of the problem was intentionally limited to certain of its aspects only. The available statistical material for these experimental calculations was too meagre and too heterogeneous to allow of really conclusive results. Any decision as to the best method of arriving at the construction of international index numbers of rents for use in international costs of living comparisons must ultimately have regard to the methods adopted for constructing index numbers of food costs, a problem which has not yet been settled. The present study has, however, brought out certain points which may be considered as established.

In the first place, it is quite clear that the statistical material at present available still suffers from too many omissions and imperfections to enable satisfactory international rent index numbers to be calculated at the present time. But it seems by no means impossible to obtain the necessary improvements, considering the progress that has been made in this field during the last few years.

With regard to rent statistics, it would be desirable if the national statistics were to follow more closely the framework of the international statistics, and if separate data could be obtained for a larger number of categories of size and comfort [1]. It should be possible to abolish the divergence in the actual nature of the data (workers' dwellings and all dwellings), so that all the data might relate only to workers' dwellings.

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[1] It should be observed that this does not mean that data should be collected for all categories of size and comfort in every town; if, for instance, the mixed method is adopted, it would be sufficient to have data for the categories of some importance in the " region " to which the town belongs. For the base towns of each region, however, the data should be more complete, so as to allow of linking up the regional indexes (by the system of double weighting).

At the same time, the statistical material on which the weights are based should be improved. If the censuses of 1930-1931 do not provide the necessary material, special enquiries, or simply reports based on a general knowledge of the special conditions of each town, should make it possible to fix for a certain period (a decade, for example), the data that ought to be known: the principal types of workers' dwellings in each town (distinguished according to the criteria of size and the presence or absence of a bathroom), and the approximate proportion of each type among all workers' dwellings, account being taken in some manner or other of the influence of the size of workers' families in the different towns.

As regards the choice of the method of weighting, the results reached here give only a rough idea of the solution that ought to be aimed at, and the question cannot yet be held to be finally settled. The rough method of the mode—an index calculated from the rent of a single type of dwelling, that predominating in each town—ought apparently to be rejected. Neither do the methods described here as those of " variable weighting " and " standard weighting " appear absolutely satisfactory. At the most, it might be possible to use the first for constructing a very approximative index as long as the available statistical material does not permit the application of more refined methods. As soon as this practical condition has been realised—and it probably could be with less difficulty than seems likely—it would appear that attention should be devoted to the method described as that of double weighting or to the " mixed " method (or to any variant or refinement of these methods).

However modest the results obtained from these researches may be, and notwithstanding all the difficulties to be overcome in attempting to construct an international rent index number, the effort appears worth continuing. For not until fairly reasonable results are obtained in this field will it be possible to make at all satisfactory international comparisons of costs of living. Those based on food costs alone, possibly including heating and lighting, cannot be other than inadequate, even using the most refined methods, for the relation borne by the cost of housing to the cost of food in one country has nothing in common with the same relation in another. If proof were necessary, it is sufficient to glance at table VII, which shows alongside the various indexes of rent, indexes of the food costs obtained by the method hitherto used by the Office. Imperfect though all these series may be, it cannot be denied that the results shown for these two groups of expenditure differ fundamentally. The inclusion of the cost of housing must therefore modify, sometimes to a substantial extent, the relations which have hitherto been compiled only on the basis of the cost of food.

The present study, it should be recalled, is only a first attempt to investigate a field hitherto almost entirely neglected, and it is hoped that by continuing researches of this kind it will be possible in due course to compile satisfactory international rent index numbers.

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